On the Calculation of the Reduction Factor for Ground Resistance of Foot

A. C. B. Alves  B. Alvarenga  E. G. Marra

Abstract - This paper is concerned about the calculation of reduction factor \( (C_f) \) of the ground resistance of the foot in contact with the gravel that covers the soil in substations yards, which is an essential parameter to determining the permissible touch and step voltages. It is proposed a new solution method for the equation published in ANSI/IEEE Std 80-2000, from the points of view of numerical and analytical. In addition to proposing alternative procedures for calculation, this article gives a comparative analysis of different expressions of \( C_f \), found in the literature focusing on the accuracy of results and the level of complexity that each expression offers in relation to mathematical operations. Numerical results of the proposed solutions are presented.

Keywords - Electric shock, safety criteria, grounding mesh design, reduction factor, contact resistance.

I. INTRODUCTION

THE first and fundamental stage in the grounding system design is the determination of the maximum permissible touch and step potentials that the human body may be subjected without risk of ventricular fibrillation (in fact, the statistical risk is 0.5%). Under the current methodology, the calculation of these maximum quantities depends on the fault time, the human body resistance (a left hand-to-foot body resistance of 1,000 \( \Omega \) is assumed), and especially the model of contact resistance of the human foot with the surface of gravel (or other material, for example, asphalt). The most significant limiting factor of the current through the body in the event of an electrical shock is the foot contact ground resistance. For modeling purposes, the feet resistances is usually connected in parallel (for touch potential calculations) or in series (for step potential calculations). Hence the importance of having accurate methods for modeling the contact resistance of the human foot, denoted by \( R_f (\Omega) \). According to [1], the ground resistance of foot is given by (1).

\[
R_f = \frac{\rho_s b}{4\pi},
\]

where \( \rho_s (\Omega \cdot m) \) is the resistivity of the surface layer, \( b (m) \) the radius of a circular plate representing the foot and \( C_f \) is the so-called reduction factor.

The modeling of the ground resistance of the foot in contact with surface layer in switchyards has suffered major changes in recent decades motivated by the incorporation of new elements to the model and the consideration of physical effects previously neglected [2]-[3]. Thus, in the design process, the initial assumption of a homogeneous soil turns into a problem with a medium where the propagation of electric current from the ground to the grid occurs. The soil is considered to have two layers: a covering material, such as a crushed rock layer, and the soil beneath this layer. Representation of the human foot in the form of a hemispherical electrode used in the 1986 edition of ANSI/IEEE Std 80 evolved into a metal circular plate having an area approximately equal to 200 cm\(^2\) (in reality, the radius is 0.08325 m and is considered as 0.08 m). The model adopted by IEEE Std 80-2000 was first published by Thapar et al [4].

The reduction factor \( C_f \) depends on the layer thickness of the covering material, \( h_s (m) \), the ratio between the gravel resistivity \( \rho_s (\Omega \cdot m) \) and the native soil resistivity \( \rho (\Omega \cdot m) \), as well as the model of the human foot touching the surface layer [3]. Although the procedure of determining \( C_f \) is complicated, to enable the correct use of this factor in computing applications of grounding system design, it is crucial that techniques of calculation satisfy the requirement of simplicity, without losing the accuracy of results.

In spite of the \( R_f \) models developed during the years 1980 and 1990, motivated by the necessity to provide engineers with simple expressions and immediate use, the reduction factor \( C_f \) is found in different versions in the literature and technical standards. \( C_f \) is then usually presented in graphic form or in mathematical expressions that aren’t always consistent with each other, sometimes contributing to obscure the issue and leading to design errors.

The aim of this work is to contribute for the clarity of this theme: methods of calculating \( C_f \) factor contained in the ANSI/IEEE Standard 80-2000 are discussed. One of these methods is based on numerical integration and the second, an approximated analytical solution of an integral. Results are obtained by these methods and comparisons with expressions published in the literature are made.

II. RELEVANCE OF THE TOPIC

The design of a grounding system starts with the initial geometric configuration, the calculation of the ground potential rise (GPR) and the determination of touch and step...
voltages [5]. The iterative process at each step redefines geometrical quantities, such as the subdivision of the mesh grid, the number of conductors and ground electrodes. The calculation reaches the convergence while the potentials obtained in the ground mesh result below the permissible touch and step voltages whose expressions are (2) and (3).

\[ V_{\text{permissible\_touch}} = (1000 + 1.5 \rho_s C_s) \cdot 0.116 \sqrt{t}, \]  

\[ V_{\text{permissible\_step}} = (1000 + 6 \rho_s C_s) \cdot 0.116 \sqrt{t}, \]  

where \( C_s \) is now the reduction factor of ground resistance of the foot in contact with the protective layer covering the substation soil, which is the subject of the present work, \( \rho_s \) (\( \Omega \cdot m \)) is the resistivity of the material, and \( t \) is the shock duration in seconds. These expressions are valid for a 50 kg person and for 0.03 \( s \leq t \leq 3.0 \) s [1].

As can be seen from (2) and (3), when \( C_s \) is assumed lower than its real value, the permissible touch and step potentials are lower, as a consequence. This will cause unnecessary spends with the grounding system.

III. REVIEW

The reduction factor \( C_s \) is approached differently in publications over the years. A short review is presented.

A. Proposition of the 1950 decade [2]-[3]

The human foot was represented by a circular conductive metal plate of negligible thickness and radius equal to \( b \) (m) touching the surface of an infinite and homogeneous soil with a resistivity denoted by \( \rho \) (\( \Omega \cdot m \)). The resistance \( R_f \) (\( \Omega \)) of the plate to remote earth is given by (4).

\[ R_f = \frac{\rho}{4b} \]  

The assumption of a homogeneous soil is not consistent with the phenomenon of conducting electrical current through the earth. Therefore, it is an inadequate model and leads to unreliable safety criteria.

B. Model of ANSI/IEEE Std 80-1986

The foot is represented in the form of a hemispherical metal electrode surrounded by a gravel layer or other material of high resistivity, forming a concentric shell of thickness equal to twice the depth of the covering surface. The radius of the equivalent hemispherical conductor is taken as 0.106 m [4]. Std 80-1986 presents graphs of \( C_s \) versus thickness \( h_s \) for a protective surface layer that covers the switchyard, which resistivity is greater than that of the underlying soil [5]. In this model, \( C_s \) is obtained from formula (5).

\[ C_s = \frac{1}{0.96} \left[ 1 + 2 \sum_{n=1}^{\infty} \frac{K^n}{1 + (2n h_s/0.106)^2} \right], \]  

where \( h_s \) (m) is the thickness of the upper layer in meters and \( K \) is the reflection coefficient given by:

\[ K = \frac{\rho - \rho_s}{\rho + \rho_s}, \]  

where \( \rho \) (\( \Omega \cdot m \)) and \( \rho_s \) (\( \Omega \cdot m \)) are the resistivity of the homogeneous soil and the resistivity of the protective layer, respectively.

As an example design, consider a homogeneous soil with \( \rho = 100 \Omega \cdot m \) and an upper layer with \( h_s = 0.1 \) m and \( \rho_s = 2,000 \Omega \cdot m \). According to the 1986 and 2000 editions of IEEE Std 80, this example has \( C_s = 0.555 \) and 0.705, respectively. In this case, the resulting increases in the permissible touch and step voltages (2)-(3) are 17% and 24%, when comparing designs based on version 2000 to designs based on version 1986, respectively [2]. The practical effect of these differences is the difficulty in the convergence of iterative process during the project steps for making the safety criteria unnecessarily rigid leading to a grounding system with more components and that needs more investment.

Equation (5) leads to errors, specially for the practical case of gravel layer of thickness in the range of 1 to 6 inches (0.0254 to 0.1524m) [2], [6]. Usually, upper layer thicknesses in grounding systems are coincidently in that range. Moreover, it has been observed by Thapar et al [4] and by Lee and Meliopoulos [6] that this model is not based on scientific accepted principles.

The same standard stated in a footnote an approximation for (5), with the aim of avoiding the infinite summation series calculation. The empirical expression (7) is then proposed [3].

\[ C_s = \frac{0.106 \left( 1 - \frac{\rho_s}{\rho} \right)}{2 h_s + 0.106} \]  

where: \( 1 - \frac{\rho}{\rho_s} = \frac{2K}{K-1} \).

Brazilian standard NBR 15751/2009 [7] adopts expressions (5) and (7), except that \( \rho \) is taken in (6) as the resistivity of the first layer of soil, obtained by a process of soil stratification. The document presents a set of curves of \( C_s \) versus \( h_s \) for positive coefficients of reflection \((K > 0)\). Kindermann & Campagnolo [8] indicate (5) and (7) for the reduction factor calculation, assuming \( \rho \) in the calculation of \( K \) coefficient as the apparent resistivity, designated by \( \rho_a \). It is probably the most accessed bibliographic source in Portuguese for specialist engineers.

C. Model of Thapar et al [4]

This model consists in the representation of the foot as a thin circular conducting plate of radius \( b \) located on the surface of the protective layer that covers the native soil. Using the method of images, this model takes into account the mutual ground resistances between the images and the plate. The expression for the reduction factor \( C_s \) is shown in (8).

\[ C_s = 1 + \frac{16b}{\rho} \sum_{n=1}^{\infty} \frac{K^n R_m (2nh_s)}{\rho_i} \]  

where \( R_m \) is the mutual ground resistance between the \( n \)-th image and the plate, and \( 2nh_s \) represent the separation for \( n = 1,2,3,\ldots \); the notation \( R_m (2nh_s) \) indicate \( R_m \) as a function of \( 2nh_s \).

In order to obtain \( R_m \) consider two circular, parallel, coaxial conducting plates \( D_1 \) and \( D_2 \) of radius \( b \) (m), both immersed in
a medium with resistivity \( \rho_s \). Take a point of coordinates \((r, z)\) located between the plates, as illustrated in Fig. 1.

Assuming that plate \( D_1 \) discharges a current \( I \) in an infinite medium of resistivity \( \rho \), and the current is not affected by the presence of \( D_2 \), and using cylindrical coordinates, \( r \) and \( z \), with \( r = \sqrt{x^2 + y^2} \), the potential at point \((r, z)\) is given by (9) [1].

\[
V(r, z) = \frac{I \rho_s}{4\pi h} \sin^{-1} \left( \frac{2b}{\sqrt{(r-b)^2 + z^2} + \sqrt{(r+b)^2 + z^2}} \right), \tag{9}
\]

where \( z = 2nh \).

The average potential on the surface of \( D_2 \) produced by the current from \( D_1 \) is determined (10).

\[
V_{D_2} = \frac{1}{2\pi} \int_0^b V(r, z) 2\pi dr, \tag{10}
\]

where \( 2\pi dr \) is a differential element of area belonging to \( D_2 \).

The mutual resistance between plates is given by:

\[
R_m(2nh) = \frac{V_{D_2}}{I} \tag{11}
\]

Combining (9), (10) and (11), the mutual ground resistance (12) is obtained.

\[
R_m(2nh) = \frac{\rho_s b}{2\pi h} \left[ 1 - \int_0^b \rho_s rsen^{-1} \left( \frac{2b}{\sqrt{(r-b)^2 + z^2} + \sqrt{(r+b)^2 + z^2}} \right) dr \right] \tag{12}
\]

Equation (12) represents the mutual resistance between plate \( D_1 \) and its images for \( n = 1, 2, 3, \ldots \). It allows obtaining \( C_s \) (8).

Numerical results for \( C_s \) obtained by (8) are presented in graphic form [4] for \( K \) ranging from \(-0.10 \) to \(-0.98 \). In the same reference, the simple empirical equation (13) is presented as an approximation to (8).

\[
C_s = \frac{1 + K}{1 - K} - \frac{4K}{\pi(1-K)} \left[ 1 - (2h/b) - 0.21K^2(e^{-7h} - e^{-30h}) \right] \tag{13}
\]

Equation (13) is applicable for \( h \) in the range of 0.0 to 0.3 m [4], [5]. According to Sverak [5], ANSI/IEEE Std 80-1996 adopts this last formulation.


This standard adopts the original model proposed by Thapar et al [4], as described by (8). In order to overcome difficulties in calculation of mutual resistances and to avoid the infinite summation series, the same document presents a simple alternative expression (14) for the \( C_s \) factor:

\[
C_s = 1 - \frac{0.09 \left( 1 - \frac{\rho}{\rho_s} \right)}{2h + 0.09}, \tag{14}
\]

where \( h \) is the thickness of the upper layer in meters.

IV. NUMERICAL SOLUTION

In order to evaluate the integration in (12) an algorithm is presented in this section, using the Simpson’s 1/3 rule [9]. For the calculation of the reduction factor, it must be settled the maximum number of terms in the series, denoted by \( n_{\text{max}} \).

Algorithm 1: \( C_s \) factor by Simpson’s 1/3 rule

Data: layer thickness \( h \), resistivity \( \rho \) and \( \rho_s \), radius \( b \) and even number of subintervals \( N \) (for example, \( N = 100 \)).

\[
K = \left( \frac{\rho}{\rho_s} \right), \frac{N+1}{2}, h = \frac{h}{h}, \text{soma} \leftarrow 0, n \leftarrow 1
\]

Repeat

\[
\text{z} \leftarrow 2nh, r \leftarrow 0, S \leftarrow f(r, b, z) + 4f(r+h, b, z)
\]

For \( i = 1, \ldots, N-1 \)

\[
S \leftarrow S + 2f(2h, b, z) + 4f((2i+1)h, b, z)
\]

\[
\text{soma} \leftarrow \text{soma} + \left( (8/\pi b^2) \times K \times S \right), n \leftarrow n + 1
\]

Until \( n \) equals \( n_{\text{max}} \)

\[
C_s \leftarrow 1 + \text{soma}
\]

In Algorithm 1, \( f(r, b, z) \) represents the integrand for the calculation of \( R_m(12) \).

V. A NEW ANALYTICAL METHOD

In addition to the numerical integration method described in algorithm 1, an approximated analytical solution of the definite integral in (12) is proposed and is shown in the following lines.

The question is to solve integral (15).

\[
I = \int_0^b \left( \frac{2b}{\sqrt{(r-b)^2 + z^2} + \sqrt{(r+b)^2 + z^2}} \right) dr \tag{15}
\]
Considering the expression \( \sin \phi = \frac{2b}{\sqrt{(r-b)^2+z^2} + \sqrt{(r+b)^2+z^2}} \) and given the parameter \( z \) and the limit of integration \( b \), it is proposed an approximation to the integral based on the known trigonometric relationship \( \sin \phi = \phi \) for \( \phi \approx 0 \).

In this approximation, \( \phi < 0.26 \text{ rad} \) (15 deg) leads to an error of less than 1.2%. This approach is much more accurate with greater thickness of the gravel layer. Using the approximation, a new form of (15) is written as:

\[ I = \int_{0}^{b} \frac{2hr}{\sqrt{(r-b)^2+z^2} + \sqrt{(r+b)^2+z^2}} dr \]  

(16)

The analytical solution of (16) leads to:

\[ I = A + B + C + D + E \]  

(17)

where:

\[ A = \frac{\sqrt{4b^2 + z^2}}{2} \]
\[ B = -\frac{h}{4} \sqrt{b^2 + z^2} \]
\[ C = \frac{1}{4} \ln \left( \frac{\sqrt{4b^2 + z^2} + 2b}{z} \right) \]
\[ D = -\frac{1}{4} \ln \left( \frac{\sqrt{b^2 + z^2} + b}{z} \right) \]
\[ E = \frac{1}{4} \ln \left( \frac{\sqrt{b^2 + z^2} - b}{z} \right) \]

It must be emphasized that (17) depends on \( n \) and \( h \), since \( z = 2nh \). Combining (8) and (17), the reduction factor is calculated by (18).

\[ C_{n} = 1 + \frac{8}{db^2} \sum_{n=1}^{\infty} K^n [A(n) + B(n) + C(n) + D(n) + E(n)] \]  

(18)

VI. RESULTS AND DISCUSSION

A. Integration by the numerical method

The Simpson’s method is implemented for the following data: \( b = 0.08 \text{m}; h = 0.025 \text{m} \) (1 inch) to 0.45m (18 inches); \( K \) in the range of 0 to –0.98; \( n_{\text{max}} = 30 \).

The reduction factors obtained by the Simpson’s rule are shown in Tables I and II.

<table>
<thead>
<tr>
<th>( K )</th>
<th>Thickness ( h_n )</th>
<th>0.025</th>
<th>0.075</th>
<th>0.100</th>
<th>0.125</th>
<th>0.150</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.1</td>
<td>0.8301</td>
<td>0.8042</td>
<td>0.8404</td>
<td>0.8993</td>
<td>0.9409</td>
<td></td>
</tr>
<tr>
<td>-0.2</td>
<td>0.7943</td>
<td>0.8562</td>
<td>0.8923</td>
<td>0.9148</td>
<td>0.9500</td>
<td></td>
</tr>
<tr>
<td>-0.3</td>
<td>0.7701</td>
<td>0.7946</td>
<td>0.8596</td>
<td>0.8777</td>
<td>0.8993</td>
<td></td>
</tr>
<tr>
<td>-0.4</td>
<td>0.7670</td>
<td>0.7914</td>
<td>0.8078</td>
<td>0.8413</td>
<td>0.8710</td>
<td></td>
</tr>
<tr>
<td>-0.5</td>
<td>0.7690</td>
<td>0.7919</td>
<td>0.8078</td>
<td>0.8419</td>
<td>0.8749</td>
<td></td>
</tr>
<tr>
<td>-0.6</td>
<td>0.7700</td>
<td>0.7940</td>
<td>0.8120</td>
<td>0.8476</td>
<td>0.8848</td>
<td></td>
</tr>
<tr>
<td>-0.7</td>
<td>0.7700</td>
<td>0.7940</td>
<td>0.8120</td>
<td>0.8476</td>
<td>0.8848</td>
<td></td>
</tr>
<tr>
<td>-0.8</td>
<td>0.7700</td>
<td>0.7940</td>
<td>0.8120</td>
<td>0.8476</td>
<td>0.8848</td>
<td></td>
</tr>
<tr>
<td>-0.9</td>
<td>0.7700</td>
<td>0.7940</td>
<td>0.8120</td>
<td>0.8476</td>
<td>0.8848</td>
<td></td>
</tr>
<tr>
<td>-0.9</td>
<td>0.7700</td>
<td>0.7940</td>
<td>0.8120</td>
<td>0.8476</td>
<td>0.8848</td>
<td></td>
</tr>
</tbody>
</table>

B. Integration by the new analytical method

The analytical solution of the integral in (12) showed in section V is used to calculate \( C \). Taking the same data used in subsection A, the results are shown in Tables IV and V.

Table III shows the influences of the subintervals number for Simpson’s rule on the accuracy of the values of \( C \). This table is obtained for \( h = 0.025 \text{m} \) and \( K = -0.9 \). It is seen by the results in this table that the required number of subinterval is not high. The results are accurate with \( N_{h} = 20 \).

<table>
<thead>
<tr>
<th>( K )</th>
<th>Thickness ( h_n )</th>
<th>0.200</th>
<th>0.250</th>
<th>0.300</th>
<th>0.350</th>
<th>0.400</th>
<th>0.450</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.1</td>
<td>0.9794</td>
<td>0.9809</td>
<td>0.9814</td>
<td>0.9821</td>
<td>0.9829</td>
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<td></td>
</tr>
<tr>
<td>-0.2</td>
<td>0.9547</td>
<td>0.9564</td>
<td>0.9594</td>
<td>0.9616</td>
<td>0.9638</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.3</td>
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<td>0.9474</td>
<td>0.9569</td>
<td>0.9668</td>
<td>0.9769</td>
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<td></td>
</tr>
<tr>
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<td>0.9165</td>
<td>0.9266</td>
<td>0.9345</td>
<td>0.9423</td>
<td>0.9503</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.5</td>
<td>0.8994</td>
<td>0.9185</td>
<td>0.9330</td>
<td>0.9447</td>
<td>0.9544</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.6</td>
<td>0.8835</td>
<td>0.8999</td>
<td>0.9121</td>
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<td></td>
</tr>
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<td></td>
</tr>
<tr>
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<td>0.8410</td>
<td>0.8515</td>
<td>0.8592</td>
<td>0.8690</td>
<td>0.8777</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE II

REDUCTION FACTORS OBTAINED BY SIMPSON’S 1/3 RULE

0.200m ≤ \( h \) ≤ 0.450m

<table>
<thead>
<tr>
<th>( K )</th>
<th>Thickness ( h_n )</th>
<th>0.33542384</th>
<th>0.33542384</th>
<th>0.33542384</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.1</td>
<td>0.9959</td>
<td>0.9959</td>
<td>0.9959</td>
<td></td>
</tr>
<tr>
<td>-0.2</td>
<td>0.9895</td>
<td>0.9905</td>
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<td>-0.3</td>
<td>0.9829</td>
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</tr>
<tr>
<td>-0.4</td>
<td>0.9764</td>
<td>0.9764</td>
<td>0.9764</td>
<td></td>
</tr>
<tr>
<td>-0.5</td>
<td>0.9700</td>
<td>0.9700</td>
<td>0.9700</td>
<td></td>
</tr>
<tr>
<td>-0.6</td>
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<td></td>
</tr>
<tr>
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</tr>
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<td>0.9521</td>
<td>0.9521</td>
<td></td>
</tr>
<tr>
<td>-0.9</td>
<td>0.9466</td>
<td>0.9466</td>
<td>0.9466</td>
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</table>

TABLE III

EFFECT OF THE NUMBER OF SUBINTERVALS ON RESULT ACCURACY

<table>
<thead>
<tr>
<th>Subintervals number ( N_{h} )</th>
<th>10</th>
<th>20</th>
<th>100</th>
</tr>
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<tbody>
<tr>
<td>0.33542384</td>
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<td></td>
</tr>
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<td>0.33542384</td>
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</tr>
</tbody>
</table>

TABLE IV

REDUCTION FACTORS OBTAINED BY PROPOSED ANALYTICAL METHOD

0.025m ≤ \( h \) ≤ 0.150m

<table>
<thead>
<tr>
<th>( K )</th>
<th>Thickness ( h_n )</th>
<th>0.025</th>
<th>0.050</th>
<th>0.075</th>
<th>0.100</th>
<th>0.125</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.9959</td>
<td>0.9959</td>
<td>0.9959</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.2</td>
<td>0.9895</td>
<td>0.9905</td>
<td>0.9905</td>
<td>0.9905</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.3</td>
<td>0.9829</td>
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<td>-0.4</td>
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<td>-0.5</td>
<td>0.9700</td>
<td>0.9700</td>
<td>0.9700</td>
<td>0.9700</td>
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<td></td>
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<tr>
<td>-0.6</td>
<td>0.9639</td>
<td>0.9639</td>
<td>0.9639</td>
<td>0.9639</td>
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<tr>
<td>-0.7</td>
<td>0.9580</td>
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<td>-0.8</td>
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<td>0.9521</td>
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</tr>
<tr>
<td>-0.9</td>
<td>0.9466</td>
<td>0.9466</td>
<td>0.9466</td>
<td>0.9466</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This result is confirmed in Fig. 2, where the reduction factor \( C \) is plotted against thickness \( h \), for the two methods. The summation series is cut off at the 30th term.
TABLE V
REDUCTION FACTORS OBTAINED BY PROPOSED ANALYTICAL METHOD

| \( K \) | \( h \mid 0.200 \text{m} \mid h \mid 0.450 \text{m} \) |
|---|---|---|---|
| \( 0.025 \) | 0.299 | 0.299 | 0.400 | 0.400 |
| \( 0.050 \) | 0.246 | 0.307 | 0.423 | 0.423 |
| \( 0.100 \) | 0.200 | 0.368 | 0.479 | 0.479 |
| \( 0.200 \) | 0.097 | 0.931 | 0.931 | 0.931 |
| \( 0.450 \) | 0.096 | 0.957 | 0.957 | 0.957 |

TABLE VII
REDUCTION FACTORS OBTAINED BY THE SIMPSON’S 1/3 RULE AND THE EXPRESSION (5) OF IEEE STD 80-1986

<table>
<thead>
<tr>
<th>( K )</th>
<th>( \text{Simpson} )</th>
<th>( \text{Expression (5)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0.025 )</td>
<td>0.894</td>
<td>0.990</td>
</tr>
<tr>
<td>( 0.050 )</td>
<td>0.924</td>
<td>0.953</td>
</tr>
<tr>
<td>( 0.100 )</td>
<td>0.971</td>
<td>0.958</td>
</tr>
<tr>
<td>( 0.200 )</td>
<td>0.563</td>
<td>0.619</td>
</tr>
<tr>
<td>( 0.450 )</td>
<td>0.555</td>
<td>0.731</td>
</tr>
</tbody>
</table>

Examination of the results in Table VII, shows that the disagreement in the values of \( C \), increases as both \( K \) and \( h \) decrease, reaching –75% (for \( K = –0.9 \) and \( h = 0.025 \) m). It is also noted that for the range 0.025 m \( \leq h \leq 0.1 \) m (usual for grounding systems), there is a large disagreement, ranging from –21% to –75%.

Fig. 3 presents graphics of the \( C \) factor versus thickness, for both editions of the ANSI/IEEE Std 80, 1986 and 2000. The results of Std 80-2000 are obtained with the Simpson’s 1/3 rule, while Std 80-1986 results are obtained by (5). In both methods, the series is cut off at the 30th term.

From the graphics of Fig. 3 it is evident that the use of model (5) may result to significant errors in calculating the permissible touch and step potentials, especially for common thicknesses, in agreement with which was pointed in [6]. Although (5) refers to the model adopted by ANSI/IEEE Std 80-1986, it is still found in bibliographies about grounding system, as in [7], [8].

Results obtained by (7) (which is an approximation of (5)) are compared to the results of the Simpson’s 1/3 rule (ANSI/IEEE Std 80-2000). The results are shown in Fig. 4.

Again taking the Simpson’s results as reference, the largest error observed in Table VI is +29.1% for expression (14) and for \( K = –0.9 \) and \( h = 0.025 \) m. Values of expression (13) in Table VI shows differences of at most 2.5%.

D. Comparison between Simpson and ANSI/IEEE Std 80-1986

Expression (5) of the model adopted by ANSI/IEEE Std 80-1986 is compared with the results obtained for the \( C \) factor by the Simpson’s 1/3 rule for the layer thicknesses of 0.025, 0.050 and 0.100m. Table VII shows the results.
As can be seen by Fig. 3 and 4, expression (7) shows better agreement with the model results ANSI / IEEE Std 80-2000 than with the model results of the 1986 edition of this standard.

VII. CONCLUSION

The safety criteria for the grounding design depends on the correct determination of the reduction factor of contact resistance of the human foot with the surface of the material used for covering the native soil in substations yards. In literature there are various formulas for calculating $C_s$ factor, some of which refer to an outdated model of ground resistance of foot. Several studies have showed that the errors are large in the comparison with the ANSI/IEEE Std-2000 recommended model.

This paper proposed two solutions to the expression that defines $C_s$, as established in [1] and [4]. The first proposition include numerical integration and do not depend on approximations or restrictions on the parameters for application: the numerical integration is based on the Simpson’s 1/3 rule. The second proposition is an approximate analytic solution of the integral, presented in section V.

The results show that the Simpson’s 1/3 rule is the best approach among those proposed by either correction of the results and numerical stability.

The new analytical method proposed is suitable to solve the expression (8) for layer thicknesses of covering material over 0.1m (about 4 inches) and as thickness increases, their results tend to agree with those obtained by the Simpson’s 1/3 rule. A notable feature about the proposed solutions is that there are no restrictions regarding the signal of the reflection coefficient and also for thicknesses greater than 0.3m.

The comparison of the $C_s$ factors obtained with Simpson’s rule and expressions (13) and (14) shows that (13), which appears in the 1996 edition of ANSI/IEEE Std 80, provides results with good accuracy for the model proposed in [4]. The results of the ANSI/IEEE Std 80-1986 ((5) and (7)) present high errors when compared to the current model, as showed by Fig. 3 and 4. Therefore, its use is not recommended.

As a suggestion to the organizations that elaborate standards, as well as to book authors about electrical groundings, it is proposed that algorithms (such as Algorithm 1) for calculation of the reduction factor are also provided.

VIII. ACKNOWLEDGMENT

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IX. REFERENCES


X. BIOGRAPHIES

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