UNCERTAINTY ANALYSIS OF A FLEXIBLE ROTOR SUPPORTED BY FLUID FILM BEARINGS

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Abstract: This paper is dedicated to the analysis of uncertainties affecting the dynamic behavior of a flexible rotor containing two rigid discs and supported by two fluid film bearings. A stochastic method has been extensively used to model uncertain parameters, the so-called Monte Carlo simulation. However, in the present contribution, the inherent uncertainties of the bearings’ parameters (i.e. the oil viscosity and the radial clearance) are modeled by using a fuzzy analysis. This alternative methodology seems to be more appropriated when the stochastic process that models the uncertainties is unknown. The analysis procedure is confined to the time domain, being generated by the envelopes of the rotor orbits. The hydrodynamic supporting forces are determined by considering a nonlinear model, which is based on the solution of the dimensionless Reynolds’ equation for cylindrical and short journal bearings. This numerical study illustrates the versatility and convenience of the mentioned fuzzy analysis. The results from the stochastic analysis are presented for comparison purposes.

Keywords: rotating machine, uncertainty analysis, fluid film bearings, fuzzy analysis.

1. INTRODUCTION

According to Meggiolaro (1996), the computational simulation of rotating machines is an indispensable resource for engineers. It allows a comprehensive understanding about the dynamic behavior of the system, considering the amount of variables involved on the problem. Thereby, a mathematical model capable of representing satisfactorily the dynamic behavior of a rotating machine is obtained by taking into account various subsystems, as follows: firstly, the subsystems that are defined by their geometry, as the shaft, drives and couplings; later, the gyroscopic effect; finally, the subsystems that are frequency and/or state dependent, such as the hydrodynamic bearings. The bearings are one of the most critical subsystems of the rotor system, influencing significantly on the performance, life, and reliability of the machine. According to Vance et al. (2010), many problems in rotating systems can be attributed to the design and application of the bearings. Thus, understanding the physical phenomena that involve the bearings is essential to improve the dynamic performance of the system.

The hydrodynamic bearings form an important class of bearings (Riul, 1988). In this case, the load is supported by a thin film of lubricant that separates the shaft from the bearing (i.e., there is no direct contact between the metal parts). Thus, this subsystem can theoretically offer infinite life, considering the rotor operating under safe dynamic conditions and clean lubricant (Vance et al., 2010). It is worth mentioning that due to the oil film the damping effect on hydrodynamic bearings is more pronounced than in rolling bearings, which is beneficial in machines that go through critical speeds during startup and stop down procedures. In this context, the analysis of possible uncertainties either in the geometry (e.g., radial clearance; due to the machining processes or damage) or in the operating conditions (e.g., oil temperature) that affect the performance of the hydrodynamic bearings is an important design issue.

Uncertainty analysis of flexible rotors has been studied by applying stochastic approaches based on the stochastic finite element method (Ghanem and Spanos, 1991). Didier et al. (2011) quantified the uncertainties effects in the response of flexible rotors based on the Chaos Polynomial theory. Koroishi et al. (2012) represented the uncertainties in the rotor parameters by using Gaussian homogeneous stochastic fields discretized by Karhunen-Loève expansion. In this work, the dynamic response of the system with uncertain parameters was characterized through Hypercube Latin sampling and Monte Carlo simulation. Lara-Molina et al. (2014) used a fuzzy stochastic finite element method to quantify the effects of high order uncertain parameters on the response of a rotating machine.
In agreement with the fuzzy approach, the present work proposes the application of a straightforward approach to simulate the dynamic response of a flexible rotor with uncertain parameters by performing a fuzzy dynamic analysis. For this purpose, the fuzzy uncertain parameters are mapped onto the model with the aid of the so-called a-level optimization (Moller and Beer, 2004). Additionally, the Differential Evolution algorithm is used to solve the optimization problem in the fuzzy analysis (Price et al., 2005). For comparison purposes, the Monte Carlo simulation combined with Latin Hypercube sampling is used in order to generate the envelope of responses of the stochastic rotor system. The choice of this stochastic solver is justified by the fact that Monte Carlo simulation has been successfully used as a reference stochastic solver to evaluate the variability of the dynamic responses (Sampaio et al., 2010). Three uncertainty scenarios are analyzed: (a) the first case is dedicated to the influence of uncertainties on the oil viscosity of both bearings of the rotating system (i.e., oil temperature); (b) the second case is associated to the introduction of uncertainties in the radial clearance of the bearings; (c) and the third case considers uncertainties introduced both in the oil viscosity and radial clearance of the bearings. As the uncertainties are analyzed only in the bearings’ parameters, the Monte Carlo simulation is directly applied to the deterministic finite element model of the rotor (Koroishi et al., 2012).

2. ROTOR MODELING

Equation (1) presents the differential equation that represents the dynamic behavior of a flexible rotor supported by fluid film bearings (Lalanne and Ferraris, 1998).

\[ \mathbf{M}\ddot{\mathbf{q}} + \left[ \mathbf{D} + \Omega \mathbf{D}_s \right] \dot{\mathbf{q}} + \left[ \mathbf{K} + \Omega \mathbf{K}_s \right] \mathbf{q} = \mathbf{W} + \mathbf{F}_u + \mathbf{F}_f \]  

(1)

where \( \mathbf{M} \) is the mass matrix, \( \mathbf{D} \) is the damping matrix (i.e., proportional damping \( \mathbf{D}_s \)), \( \mathbf{D}_s \) represents the gyroscope effect, \( \mathbf{K} \) is the stiffness matrix, and \( \mathbf{K}_s \) is the stiffness matrix resulting from the transient motion. All these matrices are related to the rotating parts of the system, such as couplings, discs, and the shaft. The vector \( \mathbf{q} \) is the generalized displacements, and \( \Omega \) is the shaft rotation speed. \( \mathbf{W} \) stands for the weight of the rotating parts, \( \mathbf{F}_u \) represents the unbalance forces, and \( \mathbf{F}_f \) is the vector of the shaft supporting forces produced by the hydrodynamic bearings. The shaft is modeled by using Timoshenko’s beam elements (finite element model) with two nodes and four degrees of freedom per node (i.e., two displacements and two rotations).

3. HYDRODYNAMIC BEARING MODEL REVIEW

In this work, the hydrodynamic supporting forces are determined by following the approach proposed by Capone (1986). This nonlinear model is based on the solution of the dimensionless Reynolds’ equation for cylindrical and short journal bearings (Fig. 1), as expressed by Eq. (2).

\[ \left( \frac{R}{L_b} \right)^2 \frac{\partial}{\partial \gamma} \left( \frac{\partial \bar{p}_b}{\partial \gamma} \right) = \frac{\partial^2 \bar{h}_b}{\partial \theta^2} + 2 \frac{\partial \bar{h}_b}{\partial \gamma} \]  

(2)

where \( \bar{p}_b = \bar{p}_b(\theta, \gamma) \) is the pressure distribution on the bearing (Eq. (3); \( \mu_h \) is the oil viscosity), \( \gamma \) is the longitudinal coordinate of the shaft center \( O_x \) (on the bearing position), \( \theta \) is the cylindrical coordinate, \( R \) is the shaft radius, \( L_b \) is the length of the bearing, and \( \bar{h}_b \) is the oil film thickness (\( \bar{h}_b = 1 - \bar{R} \cos \theta - \bar{z} \sin \theta \)). \( \bar{R} = x / C \) and \( \bar{z} = z / C \) are the coordinates of \( O_x \) along the \( X \) and \( Z \) directions, respectively; \( \bar{x} = x / (QC) \) and \( \bar{z} = z / (QC) \), where \( C \) is the radial clearance of the bearing. \([\bar{\gamma}]\) is a dimensionless value. The bearing surface is assumed to be stationary.

\[ \bar{p}_b = \frac{p_b}{6 \mu_h \Omega (R / L_b)^3} \]  

(3)

The simplifications applied to Reynolds’ equation allow its direct integration, leading to the analytical form of the pressure field (Eq. (4)); \( \bar{p}_b(\theta, \pm 0.5) = 0 \) are defined as the boundary conditions.

\[ \bar{p}_b(\theta, \gamma) = \frac{1}{8} \left( \frac{L_b}{R} \right)^2 \left[ \frac{(\bar{x} - 2 \bar{z}) \sin \theta - (\bar{z} + 2 \bar{x}) \cos \theta}{\bar{h}_b^3} \right] (4 \bar{\gamma}^3 - 1) \]  

(4)

The hydrodynamic forces \( \mathbf{F}_u \) are determined by the integration of Eq. (4) over the bearing supporting area.

\[ \mathbf{F}_u = - \frac{6 \mu_h R^3}{L_b} \int_{-1/2}^{1/2} \int_{-1/2}^{1/2} \bar{p}_b(\theta, \gamma) \left[ \frac{\cos \theta}{\sin \theta} \right] d\theta d\bar{\gamma} \]  

(5)
where $\alpha_a$ is the attitude angle defined in the Eq. (6).

$$\alpha_a = \tan^{-1}\left(\frac{\overline{z} + 2\overline{x}}{\overline{x} - 2\overline{y}}\right) - \frac{\pi}{2} \text{sign}\left(\frac{\overline{z} + 2\overline{x}}{\overline{x} - 2\overline{y}}\right) - \frac{\pi}{2} \text{sign}\left(\frac{\overline{z} + 2\overline{x}}{\overline{x} - 2\overline{y}}\right)$$  \hspace{1cm} (6)

In order to find the analytical expression for the hydrodynamic forces, the solution of the integral $G_h$, Eq. (7), is used to solve the Eq. (5) as shown below.

$$G_h = \int_{\alpha_a}^{\alpha_a + \pi} \frac{1}{1-x \cos \theta - \overline{z} \sin \theta} \cos \theta \, d\theta = \frac{2}{(1-x^2 - \overline{z}^2)^{1/2}} \left[ \frac{\pi}{2} + \tan^{-1}\left(\frac{\overline{z} \cos \alpha_a - \overline{x} \sin \alpha_a}{(1-x^2 - \overline{z}^2)^{1/2}}\right) \right]$$  \hspace{1cm} (7)

Therefore, the hydrodynamic force vector $F_h$ is given by:

$$F_h = -\rho \Omega \frac{RL}{4c^2} \left[ \frac{(\overline{z} + 2\overline{x})^2 + (\overline{x} - 2\overline{z})^2}{1-x^2 - \overline{z}^2} \right]^{1/2} \left[ \frac{3\overline{V}_h - G_h \cos \alpha_a - 2S_h \cos \alpha_a}{3\overline{V}_h + G_h \cos \alpha_a - 2S_h \cos \alpha_a} \right]$$  \hspace{1cm} (8)

where $V_h$ and $S_h$ are defined as:

$$V_h = \frac{2 + (\overline{z} \cos \alpha_a - \overline{x} \sin \alpha_a)G_h}{1-x^2 - \overline{z}^2}$$  \hspace{1cm} (9)

$$S_h = \frac{\overline{z} \cos \alpha_a + \overline{x} \sin \alpha_a}{1-(\overline{z} \cos \alpha_a + \overline{x} \sin \alpha_a)^{1/2}}$$  \hspace{1cm} (10)

4. FUZZY ANALYSIS

According to Lara-Molina et al. (2014), the uncertain parameters of rotating machines can be modeled by using fuzzy theory as an alternative approach to the stochastic methods. By using the fuzzy theory, it is possible to describe incomplete and inaccurate information. The theory of fuzzy sets was initially formulated by Zadeh (1965) to characterize vague aspects of information. Thereafter, it was developed a different approach for fuzzy sets that can be compared to the theory of possibilities to deal with the uncertainty of information (Zadeh, 1978). Both theories are connected, so that the uncertainties are modeled by means of the theory of fuzzy sets for the cases in which the stochastic process that describes the random variables is unknown (Moens and Hanss, 2011; Waltz and Hanss, 2013). The basic concepts of the fuzzy variables are revisited next.
4.1. Fuzzy variables

Let \( X \) be an universal classical set of objects whose generic elements are denoted by \( x \). The subset \( A (A \in X) \) is defined by the classical membership function \( \mu_A: X \rightarrow \{0,1\} \), shown in Fig. 2. Furthermore, a fuzzy set \( \tilde{A} \) is defined by means of the membership function \( \mu_A: X \rightarrow [0,1] \), being \([0,1]\) a continuous interval. The membership function indicates the degree of compatibility between the element \( x \) and the fuzzy set \( \tilde{A} \). The closer is the value of \( \mu_A(x) \) to 1, more \( x \) belongs to \( \tilde{A} \). 

![Fuzzy set and α-levels](image)

**Figure 2. Fuzzy set and \( \alpha \)-level representation**

Thus, the fuzzy set is completely defined by (where \( 0 \leq \mu_A \leq 1 \)):

\[
\tilde{A} = \{(x, \mu_A(x)) | x \in X\}
\]

(11)

For computational purposes, the fuzzy set \( \tilde{A} \) can be represented by means of subsets that are denominated \( \alpha \)-levels. These subsets, which correspond to real and continuous intervals, are defined by \( A_{\alpha_k} \) (Fig. 1), thus:

\[
A_{\alpha_k} = \{x \in X, \mu_A(x) \geq \alpha_k\}
\]

(12)

The \( \alpha \)-level subsets of \( \tilde{A} \) have the property:

\[
A_{\alpha_k} \subseteq A_{\alpha_l} \forall \alpha_k, \alpha_l \in (0,1]
\]

(13)

with \( \alpha_k \leq \alpha_l \). If the fuzzy set is convex for the unidimensional case, each \( \alpha \)-level subset \( A_{\alpha_k} \) corresponds to the interval \([x_{\alpha_kl}, x_{\alpha_kr}]\), shown in Eq. (14).

\[
x_{\alpha_kl} = \min \left\{ x \in X | \mu_A(x) \geq \alpha_k \right\}
\]

\[
x_{\alpha_kr} = \max \left\{ x \in X | \mu_A(x) \geq \alpha_k \right\}
\]

(14)

4.2. Dynamic models with fuzzy parameters

In this work, the dynamic model describes the behavior of the rotor by means of a set of differential equations. The relationship between the inputs \( x \) and outputs \( z \) of an specific dynamic model \( M_f \) is characterized by \( f \), which represents the set of differential equations of the model in Eq. (15).

\[
M_f : z(\tau) = f(x)
\]

(15)

Therefore, the function \( f \) maps the inputs \( x \) onto the outputs \( z(\tau) \). Thus, \( x \rightarrow z(\tau) \), where \( \tau \) is the independent variable of the dynamic response that may represent time, frequency or spacial coordinates. Considering the inputs of the model as fuzzy variables \( \tilde{x} \) or fuzzy functions \( \tilde{x}(\tau) \), the dynamic response of the system corresponds to the resulting fuzzy functions \( \tilde{z}(\tau) \). These fuzzy functions result from the mapping, thus \( x \rightarrow \tilde{z}(\tau) \).

4.3. Fuzzy dynamic analysis

The fuzzy dynamic analysis is an appropriate method to map a fuzzy input vector \( \tilde{x} \) onto the output \( \tilde{z}(\tau) \) of a numerical model by using the deterministic model given by Eq. (15). In structural analysis, the combination of
uncertainties modeled as fuzzy variables with the deterministic model based on the finite element method is denominated fuzzy finite element method. The fuzzy dynamic analysis includes two stages, as shown in Fig. 3.

In the first stage, for computational purposes, the input vector that corresponds to the fuzzy parameter is discretized by means of the α-level representation, presented in the Eq. (12) and Fig. 2. Thus, each element of the fuzzy parameters vector \( \mathbf{x} = (\tilde{x}_1, \ldots, \tilde{x}_n) \) is considered as an interval \( \mathbf{X}_{\alpha k} = [x_{\alpha k l}, x_{\alpha kl}] \), where \( \alpha_k \in (0, 1] \). Consequently, the sub-space \( \mathbf{X}_{\alpha k} \) is defined so that \( \mathbf{X}_{\alpha k} = \{\mathbf{X}_{\alpha k l}, \ldots, \mathbf{X}_{\alpha kl}\} \), where \( \mathbf{X}_{\alpha k} \in \mathbb{R}^n \).

The second stage is related to solving an optimization problem. This optimization problem consists in finding the maximum or minimum value of the output, at each evaluated \( \tau \), for the mapping model \( M_f : \mathbf{z}(\tau) = f(\mathbf{x}) \), thus:

\[
\begin{align*}
\tilde{z}_{\text{old}} &= \min_{\mathbf{x} \in \mathbf{X}_{\alpha k}} f(\mathbf{x}) \\
\tilde{z}_{\text{new}} &= \max_{\mathbf{x} \in \mathbf{X}_{\alpha k}} f(\mathbf{x})
\end{align*}
\]

(16)

where \( \tilde{z}_{\text{old}} \) and \( \tilde{z}_{\text{new}} \) correspond to the upper and lower bounds of the interval \( \tilde{z}_{\alpha k} = [z_{\alpha kl}, z_{\alpha kl}] \) in the α-level \( \alpha_k \). The set of discretized intervals \( [z_{\alpha kl}, z_{\alpha kl}] \) for \( \alpha_k \in (0, 1] \) composes the whole fuzzy resulting variable \( \tilde{z} \).

The fuzzy analysis of a transient time-domain system demands the solution of a large number of optimization problems regarding all α-level of interest for each considered time step. Each upper and lower bounds of the system analysis at a given time instant is obtained from the Differential Evolution optimization algorithm (Price et al., 2005). The output value of the transient analysis at the evaluated time-step constitutes the objective function. The inputs to this function are the uncertain parameters described previously as fuzzy, or fuzzy random variables.

5. NUMERICAL RESULTS

The proposed uncertainty analysis was applied to a horizontal rotating machine modeled by using 16 Timoshenko's beam elements, which is shown in Fig. 4. The finite element model is composed by a flexible steel shaft with 780 mm length and 12 mm diameter (\( E = 2.07 \times 10^{11} \) Pa, \( \rho = 7800 \) kg/m³, and \( v = 0.3 \)), two rigid discs \( D_1 \) (node #8) and \( D_2 \) (node #11), both of steel with 100 mm diameter and 20 mm thickness (\( \rho = 7800 \) kg/m³), and two cylindrical hydrodynamic bearings \( B_1 \) and \( B_2 \), located at the nodes #4 and #14, respectively), each one with 25 mm diameter, 10 mm length, and radial clearance of 50µm (i.e., the deterministic radial clearance). The hypothesis of short sleeve bearing is justified from the bearings dimensions. In this application, the oil viscosity is equal to 0.04 Pa.s (i.e., the deterministic oil viscosity; ISO VG 68 at 45°C). The rotating parts take into account a proportional damping \( (D_p = \beta \mathbf{K}) \) with the coefficient \( \beta = 2 \times 10^{-5} \). The effects of the coupling between the electric motor and the shaft are disregarded. Displacement sensors are orthogonally mounted (along the horizontal and vertical directions; \( X \) and \( Z \), respectively) on the bearings and discs locations to collect the shaft vibration. An unbalance of 100 g.mm at 0° applied to each disc of the rotor was considered. Fig. 5 illustrates the placement obtained in the horizontal direction of the bearing \( B_1 \) in a linear run-down condition (3200 to 100 rev/min in 30 sec). It is possible to observe that the first critical speed of the rotating machine is, approximately, 1475 rev/min.

As mentioned, three uncertainty scenarios were considered in the present contribution. The first scenario was dedicated to the investigation of the influence of uncertainties only in the oil temperature of both bearings, by changing the oil viscosity \( \mu_o \). The second one took into account only uncertainties in the radial clearances of \( B_1 \) and \( B_2 \).
Finally, the third scenario considered uncertainties in both the oil viscosity and radial clearance. In all scenarios, the measured orbits of the rotor operating at 1000 rev/min were analyzed.

Concerning the application of the fuzzy analysis, the uncertain parameters were modeled by using fuzzy triangular numbers. Thus,

$$\tilde{a} = a \left(1 - \frac{P}{100} / 1 / 1 + \frac{P}{100}\right)$$

(17)

where \(a\) represents the nominal value of the parameter and \(P\) stands for the maximum percentage of amplitude in \(a_k = 0\). In this work \(P = 5\), corresponding to the three uncertainty scenarios (i.e., 5% of variation in the triangular fuzzy variables). Additionally, the measured orbits of the rotor operating at 1000 rev/min were analyzed.

In order to solve the optimization problem associated to the described fuzzy analysis, the following parameters of the Differential Evolution algorithm were used: 10 individuals in the initial population considering the first two uncertainty scenarios (20 for the third one), 100 generations, crossover probability rate of 0.8, perturbation rate of 0.8, and the strategy for the mutation mechanism was de/rand/1/bin (Lobato et al., 2010). The instantaneous radius of the orbit measured on the disc \(D_1\), i.e., \(R_{D1} = (X^2 + Z^2)^{1/2}\), was considered as being the objective function of the minimization and maximization problems (see Eq. (16)) associated to the fuzzy analysis.

Considering the Monte Carlo simulation, the first step was to determine the number of sampling \(n_s\) to be used in the stochastic modeling. For this purpose, a sensitivity analysis has been performed (not showed here). Convergence was achieved for \(n_s\) equal to 100.

Figure 7 shows the orbits measured in the four measurement planes, considering both fuzzy and stochastic analyses, and the first uncertainty scenario (i.e., uncertainty on the oil viscosity). Note that the discs’ responses (Figs. 7c, 7d, 7e, and 7f) were affected by minor variations, while the uncertainty parameter changed significantly the orbits of the bearings (Figs. 7a, 7b, 7g, and 7h). The center of the orbits moved downward according to the decreasing viscosity. This is an expected result, since the hydrodynamic force \(F_h\) (Eq. (8)) is proportional to the oil viscosity (the force decreases with the decreasing viscosity). Additionally, it can be seen that both uncertainty analyses provided similar results (if one compares Figs. 7a with 7b, 7c with 7d, 7e with 7f, and 7g with 7h).

Figure 8 shows the orbits measured in the four measurement planes, considering both fuzzy and stochastic analyses, and the second uncertainty scenario (i.e., uncertainty on the radial clearance). As observed in Fig. 7, again the responses at the position of the discs (Figs. 8c, 8d, 8e, and 8f) were affected by minor variations as compared with the orbits of the bearings (Figs. 8a, 8b, 8g, and 8h). In this case, the center of the orbits moved downward according to the increasing radial clearance. Equation (8) shows that \(F_h\) is proportional to the inverse of the square of the radial clearances (the force decreases with the increasing radial clearance). Both uncertainty analyses provided similar results. However, some
Figure 7. Envelope of the orbits considering the uncertainty only in the oil viscosity

(--- nominal; --- lower limit / $\alpha = 0$; --- upper limit / $\alpha = 0$)
Figure 8. Envelope of the orbits considering the uncertainty only in the radial clearance
(— nominal; —— lower limit / α = 0; —— upper limit / α = 0)
Figure 9. Envelope of the orbits considering the uncertainty on the oil viscosity and radial clearance
(— nominal; — lower limit / $\alpha = 0$; — upper limit / $\alpha = 0$)
irregularities can be observed in the responses obtained at the bearings, for the case in which the fuzzy approach was applied (see Fig. 8a and 8g). Remember that an optimization process is carried out in order to determine each displacement point of the orbits, which produce these unexpected variations. Additionally, it can be seen that uncertainties in the radial clearances (Fig. 8) were able to modify significantly the rotor responses. Note that smaller changes are observed in the results obtained from the uncertainties introduced in the oil viscosity (Fig. 7).

Figure 9 shows the orbits measured in the four measurement planes, considering both fuzzy and stochastic analyses, and the third uncertainty scenario (i.e., uncertainty both in the oil viscosity and the radial clearance). In this case, the results are similar to the ones shown in Fig. 8. It is worth mentioning that these results are specific for rotating machines supported by fluid film bearings. Koroishi et al. (2012) shows that for a rotating machine supported by ball bearings the orbits remained concentric.

6. CONCLUSION

In this paper two uncertainty approaches were used to evaluate the dynamic responses of a flexible rotor supported by oil film bearings, namely fuzzy and stochastic analyses. Three uncertainty scenarios were analyzed: uncertainties on the oil viscosity, uncertainties on the radial clearance, and uncertainties on the oil viscosity and the radial clearance (all applied simultaneously to both bearings). The numerical applications show that both uncertainty approaches led to similar results. However, the fuzzy analysis seems to be more advantageous since it is able to predict the real upper and lower limits of the uncertainty envelope, which is not achieved by the Monte Carlo simulation. Finally, the proposed strategy demonstrates the relevance of introducing uncertainties in the design variables from the design perspective of rotating machinery. Moreover, the stochastic approach requires the estimation of the probability density function of the uncertain parameters. Further studies will encompass evaluations on the unbalance responses and Campbell's diagram of the rotor. An experimental verification is also scheduled.

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8. REFERENCES


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