ENHANCEMENTS OF PLANT-MODEL MISMATCH DETECTION METHODS IN MPC USING PARTIAL CORRELATION

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Abstract—One of the challenges that still needs to be overcome in order to improve the performance of the model predictive control (MPC) is its maintenance. Re-identification of the process is one of the best options available to update the internal model of the MPC, in order to improve performance. However, re-identification is costly. Researchers have proposed two different methods that are able to detect plant mismatch through partial correlation analysis. Using these techniques, instead of re-identifying all the submodels in the process, only a few inputs would have to be perturbed and only the degraded parts of the model would be updated. Nevertheless, although both approaches are efficient in detecting significant mismatch, they do not provide enough information about its magnitude. This paper presents a novel method that can detect the magnitude of mismatch. This method consists of adding offline white noise with different variances before analysing the partial correlation, in order to detect the magnitude of mismatch. A simulation case study confirms the efficacy of this new technique.

Keywords—Model Predictive Control, Bias Analysis, Plant-Model Mismatch, Partial Correlation Analysis

1 Introduction

Model Predictive Control (MPC) is an important advanced control technique for complex multivariable plants, which has been widely and successfully applied to various processes in many industries. As MPC uses a model based control strategy, its performance is related to the quality of the plant model. According to (Kano and Ogawa, 2009), there are still problems that need to be solved, in order to improve the performance of this type of advanced control, that is, increase production and reduce costs. One of them is the maintenance of the MPC. Changes in the characteristics of the process are one important factor accounting for the deterioration of the plant model (Kano and Ogawa, 2009). Hence, re-identification of the plant is the key for maintenance. Nevertheless, re-identifying the process model with a large number of inputs and outputs is costly. Detecting plant-model mismatch can reduce the cost of this re-identification, by selecting a set of submodels with more significant mismatch. In this case, re-identification of all the submodels is not necessary, but only the ones detected by the algorithm.

In 2009, Badwe et al. proposed a new method using partial correlation analysis between the model residuals and the manipulated variables, to detect plant mismatch in multivariable systems. In one of its steps, it is necessary to use dynamic models in the regression step (refer to Equations (3) and (5)), which requires performing several system identifications, as shown in Section 2. In order to develop a method more appealing to industry, Carlsson (2010) applies the conventional definition of partial correlation, which, as will be shown in Section 3, applies the least squares method. It is considered simpler to implement, as it requires less computational effort but it is still robust. In Section 3 will be demonstrated that the Carlsson’s method is in fact a particular solution to the Badwe et al.’s method.

Although both methods are able to detect the submodels with plant mismatch, they do not provide information about the magnitude of the mismatch. The paper analyzes the Badwe et al.’s and Carlsson’s methods and presents a method to check whether or not identification through bias analysis is efficient. Also, a novel method is proposed to provide additional information about the magnitude of the detected mismatch. This method is not supposed to substitute the previous ones but to be an extension of them. The next sections describe both methods and provide some tools to check if their results are valid, by analyzing mainly their bias.

2 Badwe et al.’s Method

According to Badwe et al. (2009), if the MVs are correlated between each other, a regular correlation analysis between the residuals and the MVs...
might not be proportional to the model-plant mismatch. Even if the process and the model are exactly the same for some channels, model-plant mismatch might be detected incorrectly.

Badwe et al. (2009) proposed a new method using partial correlation analysis using dynamic models in the regression step in order to overcome this limitation.

Before applying this new methodology, data from MVs (manipulated variables) and residuals are obtained with sufficient excitation. Then, the disturbance free components of the MVs are found.\[ \tilde{u}_i(k) = G_{ui} \tilde{u}_i(k) + \epsilon_{ui}(k) \] \[ \tilde{e}_{ui}(k) = u_i(k) - G_{ui} \tilde{u}_i(k) \]

The next steps of the correlation analysis using dynamic models are:

1. Evaluate \( \tilde{e}_{ui}(k) \) that is the component of the \( i \)-th input of the model, \( u_i(k) \), that is uncorrelated with the rest of MVs, \( \tilde{u}_i(k) \). That is:
\[ u_i(k) = G_{ui} \tilde{u}_i(k) + \epsilon_{ui}(k) \] \[ \tilde{e}_{ui}(k) = u_i(k) - G_{ui} \tilde{u}_i(k) \]

2. A corresponding procedure is used to estimate the component of the \( j \)-th model residual, \( \tilde{e}_{ej}(k) \), that is uncorrelated with the rest of MVs, \( \tilde{u}_i(k) \):
\[ e_j(k) = G_{ej} \tilde{u}_i(k) + \epsilon_{ej}(k) \] \[ \tilde{e}_{ej}(k) = e_j(k) - G_{ej} \tilde{u}_i(k) \]

3. Evaluate the correlation between \( \tilde{e}_{uj}(k) \) and \( \tilde{e}_{ej}(k) \). A zero value for the correlation indicates no mismatch. On the other hand, the model-plant mismatch is more significant as the correlation increases in modulus.

### 3 Carlsson’s Method

If \( G_{ui} \) is identified using a time domain structure of order \( n \) using \( N \) samples, then:
\[ \begin{bmatrix} u_i(0) \\ u_i(1) \\ \vdots \\ u_i(N-1) \end{bmatrix} = \begin{bmatrix} \Phi^T(0) \\ \Phi^T(1) \\ \vdots \\ \Phi^T(N-1) \end{bmatrix} \begin{bmatrix} \theta_{ui} \\ \epsilon_{ui}(0) \\ \epsilon_{ui}(1) \\ \vdots \\ \epsilon_{ui}(N-1) \end{bmatrix} \]
\[ u_i = \Psi \theta_{ui} + \epsilon_{ui} \]

where \( \theta_{ui} \) is the parameter vector of \( G_{ui} \) and \( \Phi(k) \) is the regressor vector related to \( \tilde{u}_i(k) \). Particularly, considering a finite impulse response (FIR) structure:
\[ \Phi^T(k) = [\tilde{u}_i^T(k) \tilde{u}_i^T(k-1) \ldots \tilde{u}_i^T(k-n+1)] \]

The least squares (LS) solution to the estimate of \( \theta_{ui} \) will be:
\[ \hat{\theta}_{ui} = (\Psi^T\Psi)^{-1}\Psi^T u_i \] \[ (8) \]

The vector of residuals is calculated by Equation (10):
\[ \epsilon'_{ui} = u_i - \Psi(\Psi^T\Psi)^{-1}\Psi^T u_i \] \[ \epsilon'_{ui}(k) = u_i(k) - \tilde{u}_i^T(k) (\Psi^T\Psi)^{-1}\Psi^T u_i \] \[ (9) \]

The corresponding procedure to compute is:
\[ \epsilon'_{ej}(k) = e_j(k) - \tilde{u}_i^T(k) (\Psi^T\Psi)^{-1}\Psi^T e_j \] \[ (11) \]

Notice that the regular correlation between \( \epsilon'_{ui}(k) \) and \( \epsilon'_{ej}(k) \) when using a FIR structure and the LS method, Equations (10) and (11), is the same analysis of the partial correlation with previous input between \( u_i(k) \) and \( e_j(k) \) showed by Carlsson (2010).

Hence, it was demonstrated here that the Carlsson’s method is a particular solution of the Badwe et al.’s method (2009), when the models used to estimate \( \hat{\theta}_{ui} \) and \( \hat{\theta}_{ej} \) are FIR structures, with its parameters estimated by the LS method.

#### 4.1 Model Bias in Correlation Analysis

The bias is defined based on the model parameters (Aguirre, 2007):
\[ b = E[\theta_{ui}] - \theta_{ui} \]
\[ (12) \]

However, considering a LTI model, \( \hat{\theta}_{ui} \) is a linear combination of \( u_i \):
\[ \hat{\theta}_{ui} = A u_i \]
\[ (13) \]

where \( A \) is called linear estimator of the model parameters. Particularly, \( A = (\Psi^T\Psi)^{-1}\Psi^T \) is a LS estimator (10).

Besides that,
\[ b = E[A u_i] - \theta_{ui} = E[A (\Psi \theta_{ui} + \epsilon_{ui})] - \theta_{ui} = E[A \Psi] \theta_{ui} + E[A \epsilon_{ui}] \]
\[ (14) \]

Therefore, the bias will not be approximately zero if \( \epsilon_{ui} \) and \( A \) are correlated. The same conclusion is valid for \( \epsilon_{ej} \).

It can be concluded that in order for this technique to give non biased results, it needs to guarantee that \( \epsilon_{ui} \) and the estimator \( A \) are uncorrelated, which means that \( \epsilon_{ui} \) is orthogonal to the plane of regressors \( \Psi \).

The same is valid for \( \epsilon_{ej} \). If these conditions are not
respected, $\hat{\theta}_u$ or $\hat{\theta}_e$ will be biased and, therefore, so will $\hat{\varepsilon}_u(k)$ and $\varepsilon_e(k)$. The geometrical interpretation of this phenomenon is shown in Figure 1.

![Geometrical interpretation of partial correlation using the LS method.](image)

The LS method provides the estimated components $\hat{\varepsilon}_e(k)$ and $\hat{\varepsilon}_u(k)$ that are orthogonal to the plane of regressors $\Psi$. In FIR structure, regressors are related only to $\bar{u}_i(k)$. Otherwise, in an equation error structure, as ARX or ARMAX, regressors consist of $\bar{u}_i(k)$ and $u_i(k)$ or $e_i(k)$. In this case, $\varepsilon_u$ and $\varepsilon_e$ can be correlated to estimator $\hat{A}$ resulting in bias.

For example, even when the noise summed at the plant output is white, parameters of the ARX model $\hat{\theta}_e$ will be biased because $\varepsilon_e$ will be correlated to the regressors. Then, it is convenient to use OE structures, applying this correlation analysis with dynamic models. Expression (15) represents the output-error (OE) structure.

$$y(k) = \frac{B(q^{-1})}{F(q^{-1})}u(k) + v(k) \quad (15)$$

$B(q^{-1})$ and $F(q^{-1})$ are monic polynomials in $q^{-1}$ and $v(k)$ is white noise. The FIR structure is a particular type of OE structure ($F(q^{-1}) = 1$).

### 4 Simulation Presentation

We carried out a simulation of a process with two inputs and two outputs controlled by an MPC, in order to show the limitations in both methods concerning the detection of the plant-model mismatch magnitude. The representation of this process $G_p(s)$ and the model used in the MPC $\hat{G}_p(s)$ are shown in (16) and (17). Noise is added to each output.

$$G_p(s) = \begin{bmatrix} 4.05e^{-6s} & 1.77e^{-6s} \\ 50s + 1 & 60s + 1 \\ 5.39e^{-6s} & 5.725e^{-6s} \\ 50s + 1 & 60s + 1 \end{bmatrix} \quad (17)$$

Figures 2 and 3 show the results of Carlsson’s and Badwe et al.’s methods, respectively, when $k = 1.5$ and the noise variance is 0.0001.
It is expected that the magnitude of the plant-model mismatch would be proportional to \(|1 - k|\), reaching its higher values at \(k = 0.5\) and \(k = 2\) and having values of approximately zero when \(k = 1\). This behavior does not occur with the correlation results shown in Figure 4, which has several curves for different online white noise variances, as they were added to \(y(k)\) in the simulations for different \(k\) values. The results for a noise variance of 0.05 show that for high noise variances, the values of the correlation are not trustworthy, given that when \(k > 1\), the values of the correlation continue negative due to the influence of the noise in the process. The second conclusion is that the values of the correlation are not proportional to the plant-model mismatch magnitude, even for small noise variance. For \(k > 1.2\) and \(k < 0.8\), the correlations are almost constant, which means that both methods are not able to detect this magnitude of gain mismatch.

5 Order of the models

In this section, a process is used to study the influence of the model order using an OE structure in the results of Badwe et al.’s method. The noise variance used in this simulation is \(\sigma = 0.01\) and the noise model to generate the colored noise \(v(k)\) is:

\[
v(k) = \frac{z}{z - 0.95}w(k)
\]

(18)

The process \(G_p(s)\) and its model \(\hat{G}_p(s)\) are shown in expressions (19) and (20).

\[
\begin{align*}
G_p(s) &= \begin{bmatrix}
50s + 1 & 60s + 1 & 50s + 1 \\
5.39e^{-4s} & 5.72e^{-3s} & 6.9e^{-3s} \\
50s + 1 & 60s + 1 & 40s + 1 \\
4.38e^{-5s} & 4.42e^{-5s} & 7.2 \\
33s + 1 & 44s + 1 & 19s + 1
\end{bmatrix} \\
\hat{G}_p(s) &= \begin{bmatrix}
6.07e^{-6s} & 1.77e^{-7s} & 3.5e^{-6s} \\
50s + 1 & 66s + 1 & 50s + 1 \\
8.09e^{-4s} & 5.72e^{-3s} & 6.9e^{-3s} \\
50s + 1 & 42s + 1 & 40s + 1 \\
4.38e^{-8s} & 4.42e^{-5s} & 11.52 \\
33s + 1 & 57s + 1 & 19s + 1
\end{bmatrix}
\end{align*}
\]

(20)

Notice that there is plant-model mismatch on all submodels.

Figure 5 shows the partial correlations using OE structure with fixed size for different lags when identifying a 3x3 model.

The orders of \(B(q^{-1})\) and \(F(q^{-1})\) are defined by the variables \(n_b\) and \(n_f\), respectively. The time delay from the B-polynomial is defined by \(n_k\).

The cross-correlation between the residuals and the estimated \(\hat{\mathbf{u}}(\hat{\mathbf{\theta}})\) is approximately zero, excluding the one related to the third input and the third output, which is almost \(-1\) for all the lags, as shown in Figure 6.

Hence, Figure 5 shows incorrectly that there is no mismatch in the pair MV3-CV3, as \(\hat{G}_{p,3}(s) = 1.6 * G_{p,3}(s)\).
One way to overcome this problem is to test different model orders, until a model with small cross-correlation values is found. The new model found has $n_b = n_f = 2$ and $n_n = 0$. Hence, Figure 7 is plotted, detecting correctly the mismatch for every pairs of inputs and outputs. The related cross-correlation are approximately zero for the lags of the nine submodels. Thus, a model in which $\hat{e}_c(k)$ or $\hat{e}_u(k)$ are not orthogonal to the plane $G_u(k)$ can lead to an erroneous analysis of model mismatch.

The same procedure is performed in this method, using correlation analysis instead of the stepwise regression. In the simulations, the online noise variance should be low enough not to interfere in the correlation results. Then, different variances of offline white noise are added artificially to the residual $\hat{e}_c(k)$, before calculating the final result of the partial correlation analysis, which is the regular correlation between $\hat{e}_c(k)$ and $\hat{e}_u(k)$. The idea of this procedure is to measure the change of the correlation results with the addition of the offline white noises, in order to determine the plant-model mismatch magnitude. The smaller is this change, the higher is the magnitude of the mismatch. We will illustrate this proposal with the simulation described next.

In this section, the process and the model used are slightly different than the ones in section 4. Two types of mismatch are present. One fixed gain mismatch is present in MV1-CV1 and the other time constant mismatch depends on $k$, which influences the value of MV2-CV1. The process and the model are represented by the transfer functions in expressions (21) and (22), respectively.

$$G_p(s) = \begin{bmatrix} 4.05e^{-6s} & 1.77e^{-6s} \\ 50s + 1 & 60k + 1 \\ 50s + 1 & 5.725e^{-6s} \end{bmatrix}$$ (21)

$$\hat{G}_p(s) = \begin{bmatrix} 4.05e^{-6s} & 1.77e^{-6s} \\ 50s + 1 & 60s + 1 \\ 50s + 1 & 5.725e^{-6s} \end{bmatrix}$$ (22)

In this scenario, after running many simulations, Figure 9 was plotted. It shows the values of the correlation for different $k$ values and offline white noise variances.

**6 Proposed New Method**

**6.1 The novel method using FIR structures**

In order to overcome the limitation cited in section 4, we propose a new method. It is inspired by Kano et al. (2010), in which white noise is artificially added to model residuals before the stepwise method is executed.

The conclusion is that raising the variance of the offline white noise, reduces the modulus of the correlations. If the derivative of each curve were calculat-
ed at the point of intersection of all curves close to \( k = 1 \) and the \( x \)-axis, the curves with lower variances of offline white noise would have the higher derivative. This could be quickly observed if a straight line were plotted for each curve using values of \( k = 0.8 \) and \( k = 1.2 \), checking the angle between this line and the horizontal line. The angle found is an approximation of the calculated derivative. Finally, curves with lower noise variances have higher values of correlation in modulus compared to the curves with higher variances. In Figure 10 we present the same information as in Figure 9, but in a different form. Instead of plotting curves for every noise variance and having \( \rho \) in the \( x \)-axis, in this new figure, a curve for every \( \rho \) is plotted, having noise variance in the \( x \)-axis. Finally, a semilogarithmic scale was used in the \( x \)-axis, in order to make it easier to visualize the results.

With the information and values from Figure 10, this paper proposes a technique to calculate the plant-model mismatch magnitude, proportional to \((1 - k)\), using the values from a curve generated from a specific \( k \).

To accomplish this result, there are two useful pieces of information that need to be extracted for every curve related to a specific \( \rho \). First, there is the value of the correlation calculated with the lower variance of white noise \((\rho_{\text{min, var}})\). The second factor is the behavior of the curve with the increase of the white noise variance. Concerning the second factor, curves with higher mismatch tend to have smaller differences between their initial correlation value \((\rho_{\text{min, var}})\) and any other correlation value generated with noise variances within a certain range \( R \). The correlation values are more influenced by the process for low noise variances. For high offline noise variances, the noise is the main factor that influences the results. \( R \) is the range of offline noise variance that corresponds to the transition from high to low correlation values in modulus, with the increase of the offline noise variance. In this case, the range of noise variance chosen was between 0.01 and 0.5.

Another variable \( \text{CorrDif}(\text{var}) \) is defined as the difference between \( \rho_{\text{min, var}} \) and the correlation related to the noise variance \( \text{var} \), which must be inside the range of \( R \).

We are going to propose a function \( M(\text{var}) \), using \( \rho_{\text{min, var}} \) and \( \text{CorrDif}(\text{var}) \) as inputs. The results of \( M \) should be proportional to the plant-model mismatch magnitude. Thus, there is no need to calculate or compute the angle or the derivative mentioned previously.

The result of a specific curve related to \( k \) is:

\[
M(\text{var}) = \frac{\rho_{\text{min, var}}}{\text{CorrDif}(\text{var}) + \varepsilon}
\]  

The term \( \varepsilon \) was added, in order to avoid having infinite or very high values for \( M \) when \( \text{CorrDif} \) is very small.

It is important to clarify that there are a lot of different functions \( M \) for the same inputs, that will give good or even better results. Nevertheless, the results obtained with expression (23) for this scenario shows the potential of this method.

Choosing the variance to be calculated and the range of variances is not a simple task. And even doing it properly, it is not guaranteed to provide significant results for a given noise variance. Hence, this paper suggests taking the average of four noise variances within the previous defined range, in order to enhance the results. This average is the magnitude of
mismatch of the submodel. The results of each variance calculated and its results are shown in Figure 11.

After calculating the average, the final result would be as in Figure 12, whose values are proportional to \(1 - k_1\), as expected.

This submodel of MV1-CV1 has a fixed gain mismatch and its results are shown in Figure 13. The fact that the \(M\) values are almost constant for every \(k\) is expected, as \(k\) has influence only in MV2-CV1.

6.2 The novel method using OE structures

The previous subsection showed a successful method, applying partial correlation analysis using the least squares method and FIR structures. Nevertheless, one question that remains is whether this same technique could be used with partial correlation analysis using OE structures. As the FIR structure is a particular type of OE structure, we are referring to the OE structures which are not FIR structures or with \(P(q)\) different than 1.

As we present next, there are some issues with the results of the simulations using OE structures. Figure 14 shows the values of the correlation for different \(k\) values and offline white noise variances using Badwe et al.’s method (similar to Figure 9). There are two values of \(k\) (1.05 and 1.45) which result in unexpected correlation values and that would influence the final results of the plant-model mismatch magnitude. Figure 15 shows the cross-correlation between the residuals and \(u\) and \(e\) when using FIR and OE structures for the first lag. The correlation for the FIR structures are close to zero for every \(k\). This behavior does not occur when using OE structure as when \(k = 1.45\), for instance, the correlation is -1, which means that the result is biased and, therefore, a new order of the model should be chosen, as shown in section 5.
In this paper, a novel method to detect the magnitude of plant mismatch using MPC is proposed, by using partial correlation with previous inputs, that use the least squares approach and FIR structures. Tests show that this technique does not substitute any method, but complements them. Partial correlation analysis using OE structures provides significant results to detect mismatch, but should not be used within the proposed technique, due to higher probability of unbiased results.

Because there are many questions still to be answered, this is not a complete solution to solve this type of problem. Future works could be conducted to address the following aspects: i) this method was only tested in a 2x2 plant with first order plus dead time submodels. What would be the results using more complex submodels and process with more inputs and outputs? ii) in the presented case study there were two types of mismatch: gain and time constant. Would this method be able to detect time delay mismatch as well? iii) is this method capable of detecting magnitude mismatch using real data?

The results presented in this paper can trigger the development of more research in this area and contribute to the enhancement of methods to detect plant-mismatch.

7 Conclusions

As we showed above, the techniques of Badwe et al. and Carlsson are able to properly detect the mismatch, but do not provide information about its magnitude. Gain mismatch of 10% and 50% might have similar results.

We also showed that Carlsson’s method is a simplification of Badwe et al.’s method, due to the fact that it uses a specific least squares approach with FIR structures instead of a general identification using OE structures. In order to get consistent analysis results applying Badwe et al.’s method, it is relevant to choose convenient model structures in order to get unbiased estimations of the dynamic model parameters.

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