SCREW-BASED MODELING OF A HUMANOID BIPED ROBOT

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Abstract — This paper presents the kinematic modelling of spatial humanoid robots (HRs) using the concept of Floating Base through the Screw Theory, and its tools: Assur’s virtual chains and Davies’s Method. The proposed approach uses the virtual chains to represent the mechanism of an HR as a spatial and parallel mechanism composed by four loops, which is complex because of its degree of freedom. Associating Davies’s Method and the virtual chains, the differential kinematic model of the spatial HR was performed. The velocities of the virtual joints were obtained using the Jacobian matrix of each virtual chain, and its linear and angular displacements were obtained through numerical methods. The velocities of the real joints were computed by means of the Davies’s Method, which can solve the inverse differential kinematics of parallel mechanisms regardless its topology or its degree of freedom. The use of the Screw Theory made it possible to obtain a kinematic representation of the HR which is independent from any fixed reference frame. The proposed approach was applied to a specific HR (Bioloid, from Robotis). The related results are presented graphically from computer simulations.

Keywords — Kinematic Model, Humanoid Robots, Screw Theory, Davies’s Method.

Resumo — Este artigo apresenta a modelagem cinemática de robôs humanoides (RHs) espaciais utilizando o conceito de Base Flutuante, a Teoria de Helicoides e suas ferramentas: cadeias virtuais de Assur e o Método de Davies. A abordagem proposta utiliza as cadeias virtuais para representar um RH como um mecanismo paralelo, espacial e composto por quatro circuitos, o qual é complexo devido aos graus de liberdade que o compõe. Com o uso das cadeias virtuais e do Método de Davies, o modelo cinemático de um HR foi obtido. As velocidades das juntas virtuais foram determinadas pela matriz Jacobiana de cada cadeia virtual individualmente; os deslocamentos lineares e angulares das juntas virtuais foram determinados por métodos numéricos. As velocidades das juntas reais foram obtidas por meio do Método de Davies, o qual é capaz de resolver a cinemática diferencial inversa de mecanismos paralelos independentemente de suas topologias e de seus graus de liberdade. O uso da Teoria de Helicoides para modelagem diferencial cinemática de robôs possibilita a obtenção de um modelo independente de coordenadas de referência. O método proposto foi aplicado a um RH específico (Bioloïd, da Robotis). Os resultados deste trabalho são apresentados graficamente a partir de simulações computacionais.

Palavras-chave — Modelo Cinemático, Robôs Humanoides, Teoria de Helicoides, Método de Davies.

1 INTRODUCTION

There are two methods commonly used to model the kinematics of robots: a first one based on the Denavit-Hartenberg (DH) convention (Denavit, 1955) and the other one based on Screw Theory (ST) (Hunt, 2000; Davidson and Hunt, 2004). While the DH method is largely used in literature, the ST approach is less known.

Rocha et al. (2011) pointed out some advantages of the ST over the DH convention. The flexibility of reference choices in the successive screw displacements method is a remarkable feature, and by a good choice of parameters, simplified model equations can be generated. Screw-based modeling presents its main advantages in differential kinematic models, in which the respective Jacobian is formed by the normalized screws of the kinematic chain (Rocha et al., 2011).

It is not common to find works on screw-based modeling of humanoid robots (HRs) in the literature despite some interesting works that use the ST to approach some humanoid characteristics. Ros et al. (2004) applied ST to analyse the human mandibular mechanics. In (Saber et al., 2006), the ST was used to design and analyse a spherical humanoid neck. Zhu et al. (2009) used ST to model the kinematics of a Steward’s platform and made an analogy by means of which they show their approach would be a novel parallel robot for rotaryhumanoid wrist. Sánchez et al. (2011) used the ST to solve the inverse kinematics of the AH1N1 HR (Sánchez et al., 2011). In spite of solving the inverse kinematics using ST, the authors solved the direct kinematics by the DH convention and nothing was said about the differential kinematics of the humanoid.

In the works of Moro et al. (2011) and Moro et al. (2012), the biped robots are modeled using the DH parameters and, thus, a gait, for example, would be performed by switching between two models depending on which foot is supporting the robot (Toscano et al., 2011).

Man et al. (2007) presented a kinematic analysis of a humanoid structure by the ST. Although the authors show how to use the ST for solving the direct and inverse kinematics of an HR, and indicate the use of the screw-based Jacobian to solve its differential kinematics, their work lacks clarity how use the ST for RHs modeling.

Therefore, this work aims to present how a full body HR can be modeled using ST and its tools in a systematic and clearer way. For that, the virtual chains (Santos et al., 2006) are used to represent the HR as a closed mechanism, and it is shown how to apply the
Davies’s Method (Campos et al., 2005; Santos et al., 2006) to HRs.

This paper is organized as follows. First, the ST and its tools are briefly presented. Then, it is shown how to model the kinematics of an HR using virtual chains and the Davies’s Method. After that, the simulation results are shown. Finally, the conclusions of this work are presented.

2 SCREW-BASED KINEMATIC TOOLS

In order to simplify the comprehension and the development of the HB model, some fundamentals are briefly presented in this section.

2.1 Successive Screw Displacement Method

Charle’s theorem states that the general spatial displacement of a rigid body is a rotation about an axis and a translation along the same axis; such combination of translation and rotation is called a screw displacement (Bottema and Roth, 1979).

\[
\begin{align*}
\hat{p}_2 &= A(\theta, t) \hat{p}_1, \\
\text{in which } A(\theta, t) &= \text{the homogeneous transformation } 4 \times 4 \text{ matrix (see (Tsai, 1999) for more details).}
\end{align*}
\]

2.1.1 Successive Screw Displacements

A kinematic chain is composed by rotational and prismatic joints numbered from 1 to n. Each joint describes the displacement between two successive links starting from the base frame, defined as the Link 0, until the last link of the kinematic chain, defined as Link n. Thus, the joint \(i\) describes the relative displacement between the links \(i\) and \(i-1\).

The position and the orientation of a point on a Link \(k\) with respect to a Link \(c\) of the same considered chain can be given by (Simas, 2008):

\[
A^r_k = \bar{A}^r_c \left( \prod_{i=}^{k} A^r_{hi} \right) \bar{A}^r_{ki},
\]

where \(r\) indicates the link in which a coordinate system is fixed; \(\bar{A}^r_k\) is the homogeneous matrix that describes the position and orientation of a point on the Link \(k\) with respect to a coordinate system attached to the Link \(c\); \(\bar{A}^r_c\) is the homogeneous matrix defined in the initial position of the chain that describes the position and orientation of the reference coordinate system attached to Link \(r\) with respect to the coordinate system on Link \(c\); \(\bar{A}^r_{hi}\) is the homogeneous matrix also defined in the initial position of the chain that describes the position and orientation of the coordinate system of a point on Link \(k\) with respect to the reference on Link \(r\); and \(\bar{A}^r_{ki}\) is the homogeneous matrix that describes the relative displacement between the links \(i\) and \(i-1\) described by the screws defined with respect to the reference coordinate system attached on Link \(r\).

2.2 Screw Representation of Differential Kinematics

The complete displacement of a rigid body by means of a rotation and a translation with respect to the same axis is called screw movement (or twist) and is denoted by \(S\). The ratio of the linear and angular velocities - \(\tau\) and \(\omega\), respectively, and the latter being relative to the considered reference frame - is called pitch of the screw and denoted by \(h = \frac{\text{parallel}}{\text{tan}}\).

The twist \(S\) is composed by a pair of vectors as \(S = [\omega^T \, v_p^T]^T\), in which \(v_p\) represents the linear velocity of a point \(P\) attached to the body, which is instantaneously coincident with the origin \(O\) of the reference frame (Tsai, 1999). The twist \(S\) is completely defined by the vectors \(s\) and \(s_0\) and the scalar pitch, and the twist can be decomposed into its normalized screw \(\hat{S}\) and its magnitude \(\hat{q}\) as (Davidson and Hunt, 2004):

\[
\hat{S} = \begin{bmatrix} \omega \\ v_p \end{bmatrix} = \begin{bmatrix} s \\ s_0 \times s + \hat{q}h \end{bmatrix} \hat{q} = \hat{S}q. \tag{4}
\]
In a kinematic chain, the relative velocity between any two links is obtained by the sum of the twists of the joints between them (Rocha et al., 2011). The sum is possible only if all twists are defined according to the same referential frame. The twist can be expressed according to any reference by means of the screw transformation \( T_j^i \in \mathbb{R}^{6\times6} \) (Eq. (5)), in which \( \hat{p}_j^i \) is the skew-symmetric matrix composed by the elements of the position vector \( p_j^i \), and \( R_j^i \) expresses the orientation of the frame \( j \) with respect to the frame \( i \) (see Tsai, 1999) for more details).

\[
T_j^i = \begin{bmatrix} \hat{p}_j^i & R_j^i \\ p_j^i R_j^i & R_j^i \end{bmatrix}
\]  

(5)

### 2.3 Davies Method

Davies’s Method is a systematic manner to solve the differential kinematics of closed chain mechanisms. Davies (1981) derived his method from the Kirchhoff’s circulation law for electrical circuits. Considering a closed chain mechanism composed by \( n \) joints and in which velocity of one of its links with respect to itself is null (Campos et al., 2005; Santos et al., 2006), then, the circulation law may be expressed as:

\[
\sum_{i=1}^{n} \hat{s}_i \dot{q}_i = 0,
\]

(6)

in which \( \hat{s}_i \) is the normalized twist, \( \dot{q}_i \) is the magnitude of \( s_i \) and \( 0 \) is a null vector whose dimension is equal to the \( \hat{s}_i \). Equation (6) is the constraint equation and it can be generalized as:

\[
N\dot{q} = 0,
\]

(7)

where \( N = [\hat{s}_1 \hat{s}_2 \ldots \hat{s}_n] \) is the network matrix containing the normalized screws whose signals depend on the screw definition in the circulation law orientation, and \( \dot{q} = [\dot{q}_1 \dot{q}_2 \ldots \dot{q}_n]^T \) is the magnitude vector.

In a closed kinematic chain, primary and secondary joints have to be defined to handle passive and actuated joints, respectively. Using the constraint equation Eq. (7), the secondary joint velocities can be calculated as function of the primary joint velocities (Davies, 1981; Campos et al., 2005; Santos et al., 2006). To this end, the same equation earlier cited is rearranged in a manner to highlight the primary and secondary joint velocities as:

\[
[N_p \ N_s] [\dot{q}_s \ \ 
\dot{q}_p] = 0,
\]

(8)

where \( N_p \) and \( N_s \) are the primary and secondary network matrices, respectively, and \( \dot{q}_p \) and \( \dot{q}_s \) are the corresponding primary and secondary magnitude vectors, respectively. Thus, if the \( N_s \) network matrix is square and invertible, the secondary joint velocities may be determined as:

\[
\dot{q}_s = -N_s^{-1}N_p \dot{q}_p.
\]

(9)

### 3 KINEMATIC MODELING OF AN HUMANOID ROBOT USING SCREW THEORY

In the previous section, it was briefly presented the basis of ST and its tools that will be used in the current section to model the HR Bioloid, from Roboliv.

#### 3.1 Bioloid Humanoid Robot

Bioloid is an HR composed by one floating base (FB) (see Papadopoulos and Dubowsky (1991), Sentis (2007) and Mistry et al. (2008) for more details) link and four serial chains that mimic the four limbs of the human body. It weights 1.7 Kg and is 0.397 m in tall, and its structure is composed of eighteen actuated joints: three degree of freedom (DoF) in each arm and six DoF in each leg.

A 3D CAD model of an humanoid structure is depicted in Fig. 2. The waist is the local reference on the humanoid kinematic chain, that is, the position of the arms and the legs are given with respect to it.

![Figure 2: 3D Model of an humanoid structure.](image)

The mass and length of the links of the humanoid structure and the position of the shoulders and the thigh, with respect to the waist can be found in Toscano et al. (2011). For the sake of simplicity, the position of the center of mass of each link is right in the middle of itself. However, the position of the center of mass of the trunk of the robot, with respect to its waist, is given by the position vector \( p_{CM}^w = [-0.014563 \ 0 \ 0.86397]^T \).

#### 3.2 Modeling of The Bioloid Robot Using Screw Theory

In this section, the kinematic modeling of the Bioloid robot by the ST and its tools will be presented. Because of the bilateral symmetry of the HR structure and because the kinematic modeling is a systematic method, just the modeling of one leg will be addressed here: to design the models of the three other limbs, the procedure just has to be repeated with the parameters of each limb.

#### 3.2.1 Screw-based Modeling of One Leg

The topology of a kinematic chain that represents a humanoid leg can be of different types, depending on
the kind of movements that is expected from the chain.
In the case of this paper, the chain of the considered leg
has six actuated joints and three links. The distribution
of the joints in the kinematic chain and its links can be
seen in Fig. 3.

![Figure 3: Kinematic chain that represents one leg: (a) identification of the screw parameters; (b) three reference frames allocated in the leg.](image)

Once the screw parameters are identified in Fig. 3a, the screw of each joint can be determined like Eq. (4):

\[ \bar{s}_L = \hat{s}_L q_L, \]

in which \( L_i \) is the joint \( i \) of the leg and \( q_L \) is the mag-
nitude of the angular velocity of the joint \( i \). The Jacobian matrix of the kinematic chain representing one leg, il-
ustrated by Fig. 3, is

\[ J_L = [\hat{s}_L \hat{s}_L \hat{s}_L \hat{s}_L \hat{s}_L \hat{s}_L]. \]

As \( J_L \) is a square matrix, it can be inverted to compute the inverse differential kinematic of this chain.

**Direct Kinematics of One Leg**

The position and orientation of an end-effector in a serial kinematic chain can be determined using Eq. (3). In Fig. 3b, it was assumed that the thigh, calf and foot links are labeled as \( L_{11}, L_{12} \) and \( L_{13} \), respec-
tively. The base frame (frame 0) is allocated in the intersection point among the screw axis of the joints \( q_1, q_2 \) and \( q_3 \). The end-effector frame, called here as foot frame, is allocated in a point collinear to the link \( L_{13} \) on the sole of the foot.

![Table 1: Screw parameters of the leg shown in Fig. 3a.](image)

<table>
<thead>
<tr>
<th>Joint</th>
<th>( s )</th>
<th>( q_L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 0 1</td>
<td>( s_{01}, s_{02}, s_{03} )</td>
</tr>
<tr>
<td>2</td>
<td>1 0 0</td>
<td>( s_{02}, s_{03}, s_{04} )</td>
</tr>
<tr>
<td>3</td>
<td>0 1 0</td>
<td>( s_{03}, s_{04}, s_{05} )</td>
</tr>
<tr>
<td>4</td>
<td>0 1 0</td>
<td>( s_{04}, s_{05}, s_{06} )</td>
</tr>
<tr>
<td>5</td>
<td>0 1 0</td>
<td>( s_{05}, s_{06}, s_{07} )</td>
</tr>
<tr>
<td>6</td>
<td>1 0 0</td>
<td>( s_{06}, s_{07}, s_{08} )</td>
</tr>
</tbody>
</table>

It is necessary to allocate another reference frame that will be used to determine the matrix \( \bar{A}_r^\theta \) and \( \bar{A}_r^k \) of Eq. (3). Allocating the reference frame (called \( \text{ref} \)) in the middle of link \( L_{12} \) (Fig. 3b), the vectors \( s_0, q_L \) are determined with the new reference as:

\[ s_{0_{11}} = s_{0_{12}} = s_{0_{13}} = \begin{bmatrix} 0 \\ 0 \\ \frac{L_{11} + L_{12}}{2} \end{bmatrix}, \]

\[ s_{0_{14}} = \begin{bmatrix} 0 \\ 0 \\ L_{12} \end{bmatrix}, \]

\[ s_{0_{15}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \]

The \( \bar{A}_r^\theta = \bar{A}_r^s \) and \( \bar{A}_r^k = \bar{A}_r^l \) are, respectively, the position and orientation of the frame \( \text{ref} \) with respect to the base frame, and the position and orientation of the frame foot with respect to the frame \( \text{ref} \):

\[ \bar{A}_r^\theta = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \]

\[ \bar{A}_r^k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \]

The matrices \( A_r^{\theta,\text{ref}} \) are the matrices formed by the screw of each joint of the leg in respect to the frame \( \text{ref} \) (Fig. 3) using Rodrigues’s formula as in Tsai (1999).

### 3.2.2 Differential Kinematics of the Whole Humanoid Body

On the earlier sections, it was demonstrated how the ST can be used to model the kinematics of a chain that represents a humanoid leg. After modeling the three other limbs, it is necessary to represent all four chains at the same reference frame in the humanoid structure to have a full-body kinematic model of a HR.

The waist is the local reference frame to the humanoid structure and, thus, the reference frame to the chains of the humanoid limbs. As the limbs were modeled individually and with respect to the inertial reference frame, a transformation had to be applied to represent, for example, the right leg with respect to the waist of the humanoid. The transformation needed is made by the Eq. (5) as illustrated in Fig. 5.

The matrix \( T_{RL}^w \) “transforms” a generic leg (Fig. 3) into the right leg of the humanoid and it is composed of the skew-symmetric matrix \( \hat{P}_{RL}^w \) and the rotation matrix \( R_{RL}^w \) (Eq. (5)). The same can be done to the right arm, that is, the matrix \( T_{RS}^w \) would “transform” a generic arm into the right arm of the robot; and it would be composed of the skew-symmetric matrix \( \hat{P}_{RS}^w \) and the rotation matrix \( R_{RS}^w \). Because of the
Figure 4: Directed graph (digraph) representation of the parallel mechanism composed by the humanoid’s and the virtual chains, with the designation of the number of circuits, and its directions. $S_{ijk}$ and $V_{ijk}$ are the twists of the real and virtual joints of each limb: $i \in \{R,L\}$ for right and left side; $j \in \{L,A\}$ for leg and arm; and $k = 1,\ldots,6$. $F_{Bk}$ are the floating base twists.

Figure 5: Reference frame transformation: (a) transformation to the right leg; (b) transformation to the right arm.

bilateral symmetry of the human body, the determination of the left leg and left arm chains were done in a similar way as it was done to the right ones. That is, the transformation $T_{wL}$ would make a generic leg (Fig. 3) be the left leg and, likewise, the transformation $T_{wS}$ would make a generic arm be the left arm.

The matrices $R_{RL}$, $R_{RS}$, $R_{LL}$, and $R_{LS}$ are $3 \times 3$ identity matrices since there is no change among the orientations of the frames (Fig. 5). However, the vectors $p_{RL}$, $p_{RS}$, $p_{LL}$, and $p_{LS}$, that compose the skew-symmetric matrices, are determined by the data found in Toscano et al. (2011) (see Tsai (1999) for more details).

By placing the four limbs with respect to the humanoid waist in the robot structure, the whole HR body is formed as it can been seen in Fig. 2. It is composed of one FB allocated in its waist (that is, an 3P3R virtual kinematic chain attached between the inertial reference frame and the waist frame) and four serial kinematic chains that work together to perform a task, for example, making a gait or manipulating an object.

Using the DH parameters to solve differential kinematics of the HR, there would be five Jacobian matrices to be considered: the first between the inertial reference frame and the humanoid’s waist, and the others four, for each limb, would be with respect to the waist of the robot.

Though, by the use of the virtual chains, an HR structure can be represented as a closed mechanism and, then, the Davies’s Method can be applied, by the ST, to solve its differential kinematics. With the use of the virtual chains, the motion planning can be done in the inertial reference frame as planning motion to five collaborative robots.

The Bioloid robot has just three joints in each arm. However, in order that all matrices have appropriate sizes to the Davies’s Method, three more joints were added to each arm to solve the differential kinematics of the HB Bioloid; that is why the number of “real” joints between Figures 2 and 4 are different.

As the motion is spatial, five 3P3R virtual chains are used: one to be the FB in the biped waist and the other four are linked, each one, to the feet and to the hands. The resulting mechanism is a complex spatial parallel mechanism with four loops composed of fifty four joints and fifty one links, as it can be seen in Fig. 4 presented by the directed graph (digraph) notation (Campos et al., 2005; Tsai, 2001).

Now, with the HR represented as a closed mechanism, it is possible to apply the Davies’s Method to solve the differential kinematics for the whole body at once. First, the network matrix $N$ has to be assembled with the normalized screws. As the mechanism has fifty four joints and four loops, therefore, the matrix $N$ has dimension $24 \times 54$, and the magnitude vector has dimension $24 \times 1$ (see Appendix A for more information about the magnitude vector $q$ and the network matrix $N$).

As the method states, it is necessary to identify and distinguish the primary and the secondary joint variables in the magnitude vector. As this work uses the virtual chains to impose motion to the "real"
chains, then, the joints of the virtual chains were chosen to be the primary velocities. That means,

\[
\dot{\mathbf{q}} = \begin{bmatrix} \dot{\mathbf{q}}_s^T & \dot{\mathbf{q}}_p^T \end{bmatrix}^T,
\]

in which \(\dot{\mathbf{q}}_s\) is formed by the first twenty four lines of the vector \(\dot{\mathbf{q}}\) ("real" joint velocities) and \(\dot{\mathbf{q}}_p\) is assembled by the last thirty lines (virtual joint velocities). Choosing \(N_p\) and \(N_s\) coherently with \(\dot{\mathbf{q}}_s\) and \(\dot{\mathbf{q}}_p\), respectively, the Davies’s Method can be applied as it states in Eq. (7) and differential kinematics of the whole body was solved.

\section{Results}

To validate the screw-based model of the HR, a gait was elaborated and performed by computer simulation. In Fig. 6, it can be seen the humanoid in four different video frames during the simulation of the gait: beginning/ending of a gait cycle.

Using the FB, it was possible to model the kinematics of an HR detached from any fixed reference frame. By the use of the virtual chains, the gait was elaborated as generating trajectories for five different manipulator robots, and the trajectories were all generated in the inertial reference frame.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6.png}
\caption{Video frames of the HR while walking.}
\end{figure}

In Fig. 7, it can be seen the angular displacement of the joints of the right and left legs. The computation of those values were made by numerical method, however, Fig. 6 was generated using the screw-based direct kinematics as shown in this work.

With the use of the virtual chains, the HR could be represented as a closed mechanism and, then, the Davies’s Method was applied. In Fig. 8, it can be seen the angular velocities of the joints of the right and left legs of the HR. Those values were all computed at once through the use of the Davies’s Method.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure7.png}
\caption{Angular displacements, in rad, of the joints of the legs.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure8.png}
\caption{Angular velocities, in rad/s, of the joints of the legs.}
\end{figure}

The angular displacements and velocities of the joints of the right and left arms were computed in the same manner despite the fact they are not shown in this article.

\section{Conclusion}

This paper presented in a systematic way how to use the Floating Base concept, the Screw Theory and its tools to model a full-body humanoid robot. With the Floating Base concept, the HR could be modeled in such a way that it has only one kinematic model independently of the supporting leg. Moreover, by the use of the virtual chains, it was possible to represent a humanoid structure as a closed mechanism and, then, apply the Davies’s Method to solve the inverse differential kinematics of the whole humanoid body at once without any consideration about the topology of the robot.

By this method, to generate a trajectory for humanoid robots is almost like to generate trajectories for a multi-robot system, since the virtual chains can be understood as five different serial robots working together to perform a task. This approach allows to use various methods and tools of the collaborative robotics in humanoid robots. The trajectory generated was evaluated by the ZMP criterion (see Appendix B for more details), that made it possible to imply the stability of the simulated gait (dynamic balanced gait).
For future works, there will be a continuous study in how the tools of the Screw Theory may contribute to humanoid robotics field. It would be very interesting and promising to analyze the biped robot including the static and dynamic models of the humanoid robot also by the Screw Theory.

Appendix A - Magnitude Vector \( \mathbf{q} \) and the Network Matrix \( \mathbf{N} \)

In short, the Davies’s Method computes the magnitude of secondary joints velocities as function of the primary ones of closed chain mechanisms composed of one or more loops. However, for the algorithm to be executed, the matrix \( \mathbf{N} \) (Eq. (9)) has to be invertible. Therefore, the primary joints velocities have to be chosen in such a way to \( \mathbf{N} \) be invertible.

Let \( \dot{q} \) be the joint velocity for both rotational and prismatic joints. The actuated joint were chosen to be the secondary joints velocities, which will be calculated by means of the joints velocities of the virtual chains, the primary ones. The magnitude vector \( \hat{\mathbf{q}} \) was chosen and, then, the network matrix \( \mathbf{N} \) was assembled. Using the notation of Fig. 4, the magnitude vector - Eq. (7) - considered in this work is given by:

\[
\begin{align*}
\dot{\mathbf{q}} &= \begin{bmatrix}
\dot{q}_{RL1} & \dot{q}_{RL2} & \dot{q}_{RL3} & \dot{q}_{RL4} & \dot{q}_{RL5} & \dot{q}_{RL6} & \ldots \\
\dot{q}_{LL1} & \dot{q}_{LL2} & \dot{q}_{LL3} & \dot{q}_{LL4} & \dot{q}_{LL5} & \dot{q}_{LL6} & \ldots \\
\dot{q}_{RA1} & \dot{q}_{RA2} & \dot{q}_{RA3} & \dot{q}_{RA4} & \dot{q}_{RA5} & \dot{q}_{RA6} & \ldots \\
\dot{q}_{LA1} & \dot{q}_{LA2} & \dot{q}_{LA3} & \dot{q}_{LA4} & \dot{q}_{LA5} & \dot{q}_{LA6} & \ldots \\
\dot{q}_{FB1} & \dot{q}_{FB2} & \dot{q}_{FB3} & \dot{q}_{FB4} & \dot{q}_{FB5} & \dot{q}_{FB6} & \ldots \\
\dot{q}_{RL3} & \dot{q}_{RL2} & \dot{q}_{RL3} & \dot{q}_{RL4} & \dot{q}_{RL5} & \dot{q}_{RL6} & \ldots \\
\dot{q}_{AL1} & \dot{q}_{AL2} & \dot{q}_{AL3} & \dot{q}_{AL4} & \dot{q}_{AL5} & \dot{q}_{AL6} & \ldots \\
\dot{q}_{RA1} & \dot{q}_{RA2} & \dot{q}_{RA3} & \dot{q}_{RA4} & \dot{q}_{RA5} & \dot{q}_{RA6} & \ldots \\
\dot{q}_{LA1} & \dot{q}_{LA2} & \dot{q}_{LA3} & \dot{q}_{LA4} & \dot{q}_{LA5} & \dot{q}_{LA6}
\end{bmatrix}^T, \\
n &= (17)
\end{align*}
\]

Now, the network matrix can assembled accordingly to Fig. 4, in which can be seen the directed graph (digraph) notation of the kinematic struture of humanoid robot with the virtual chains. A graph can be understood as a set of vertices (or nodules) with edges (or lines) connecting two or more vertices; when the graph has arrows instead of edges (or lines), it is a digraph (Campos et al., 2005).

Using the graph notation to represent the kinematic structure of a mechanism (or robot), the vertices and edges correspond, respectively, to the links and joints of the mechanism. The arrows, of the digraph representation, indicate the relative motion between consecutive links (for more information about graph notation applied to mechanisms and robots, see (Tsai, 2001)).

A parallel mechanism can be represented by the digraph \( D \), which is composed of \( f \) loops and \( n \) arrows. The arrows correspond to the twists - all defined on the same reference frame - of the considered mechanism, and \( \lambda \) is the degree of freedom of the mechanism’s workspace. Its network matrix \( \mathbf{N} \) has the form:

\[
\mathbf{N}(f \times n) = \begin{bmatrix}
\mathbf{N}_1(k \times n) \\
\mathbf{N}_2(k \times n) \\
\vdots \\
\mathbf{N}_k(k \times n)
\end{bmatrix},
\]

where \( \mathbf{N}_k \), for \( k = 1, \ldots, l \), contains all the normalized twists (Eq. (4)) of the loop \( k \) in a particular order.

Considering \( A_k \) the set that contains all twists of the loop \( k \), \( \mathbf{0} \) a null column vector in \( \mathbb{R}^n \), \( OL_k \) the orientation of the circulation law (Davies, 1981) for loop \( k \), and \( OA_i \), the orientation of the arrow of the twist \( i \), the columns of the submatrix \( \mathbf{N}_k \) of Eq. (18) can be defined as:

\[
\mathbf{N}_k(i) = \begin{cases}
\mathbf{0}, & \text{if } S_i \notin A_k \\
\hat{S}_i, & \text{if } S_i \in A_k \text{ and } OL_k = OA_i \\
-\hat{S}_i, & \text{if } S_i \in A_k \text{ and } OL_k \neq OA_i,
\end{cases}
\]

(19)

Appendix B - Zero-Moment Point

Support polygon is a convex polygon determined by the contact surface between the robot’s feet and the ground where the robot walks on (Vukobratovic and Borovac, 2004).

The Zero-Moment Point (ZMP), introduced by Vukobratovic and Stepanenko (1972), is a parameter that can be used to analyze the stability of a gait, because it analyzes the contact conditions between the robot’s feet and the ground.

Consider a point on the support surface where the reaction force acts from the ground to the robot. If that point is within the support polygon, it is the ZMP, and the horizontal components of the reaction momentum are null, thus, the robot will not overturn with respect to its feet (Vukobratovic and Stepanenko, 1972; Vukobratovic and Borovac, 2004). That is, the robot will not fall down during a gait if the ZMP is within the support polygon.

The linear inverted pendulum (LIP-3D) approach proposed by Kajita et al. (2003), which consider the dynamics of the whole humanoid robot body just by the dynamics of its center of mass, gives simplified equations to determine the ZMP. For that reason, the LIP-3D approach was used to computed the position of the ZMP in this work. As proposed in Kajita et al. (2003), the ZMP is given by:

\[
\begin{align*}
P_{ZMP_x} &= P_{CoM_x} - \frac{P_{CoM_y} - P_{CoM_z}}{g} \cdot \hat{p}_{CoM_x}, \\
P_{ZMP_y} &= P_{CoM_y} - \frac{P_{CoM_z}}{g} \cdot \hat{p}_{CoM_y},
\end{align*}
\]

where \( P_{CoM} = \begin{bmatrix} P_{CoM_x} & P_{CoM_y} & P_{CoM_z} \end{bmatrix}^T \) is the position vector of the center of mass of the robot and \( g = \begin{bmatrix} 0 & 0 & -g_z \end{bmatrix} \) is the gravity vector, both with respect to the inertial reference frame.
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References


